VSB – TECHNICAL UNIVERSITY OF OSTRAVA
FACULTY OF ECONOMICS

DEPARTMENT OF FINANCE

Determinace kreditního rizika u portfolia dluhových aktiv
Determination of Credit Risk for Debt Assets Portfolio

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1. Introduction
2. Description of the Financial Risk
3. Description of the Credit Risk Management and Models
4. Determination of Credit Risk by Selected Models
5. Conclusion

Bibliography
List of Abbreviations
Declaration of Utilisation of Results from the Diploma Thesis
List of Annexes
Annexes

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The declaration

"I hereby declare that I have elaborated the entire thesis including annexes myself."

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Contents

1 Introduction

Along with the deepening of global financial innovation and development of morden banking, risk management becomes more and more complicated and important. Among all the financial risks, credit risk is the most frequent and important risk so that not only individual investors and corporates but also banks need to pay more attention. To measure credit risk, scholars and researchers from all over the world contribute to conducting an effective method. Nowadays, one popular measurement technique of credit risk is CreditMetrics™ model with the help of VaR, developed by J.P.Morgan. Moreover, Basel Accords from Basel Commitee play an important role in credit risk measurement and management as well.

The main objective of this thesis is to estimate the economic capital of ten selected bonds portfolio under CreditMetrics™ model and capital requirement for unexpected losses from credit risk under Basel Accord. It gives a possible way to compare the results from Basel Accords, including Basel I, Basel II and Basel III, and from CreditMetrics™ model.

The whole thesis can be divided into five chapter. Chapter 2 and Chapter 3 constitute therotical part. Practical part can be found in Chapter 4 and Chapter 5 is structured on summary and conclusion of the results.

Therotical part mainly focus on different types of financial risks firstly and then description of credit risk management and models. Financial risks including credit risk, market risk, operational risk and liquidity risk are described in details with some examples. Later, several models for credit risk management are introduced and CreditMetrics™ model is emphasized. At last, there is a desciption of different version of Basel Accords on capital adequacy.

In practical part, the example of a portfolio of ten selected bonds traded on Frankfurt Stock Exchange is used to determine economic capital by using CreditMetrics™ model. Furthermore, the capital requirement to cover unexpected losses is estimated by different approaches under different version of Basel Accords. The nominal value of whoe portfolio is 10 million euro and time horizon we selected is one year. And then, all the results are analyzed and compared specifically.

2 Description of financial risk

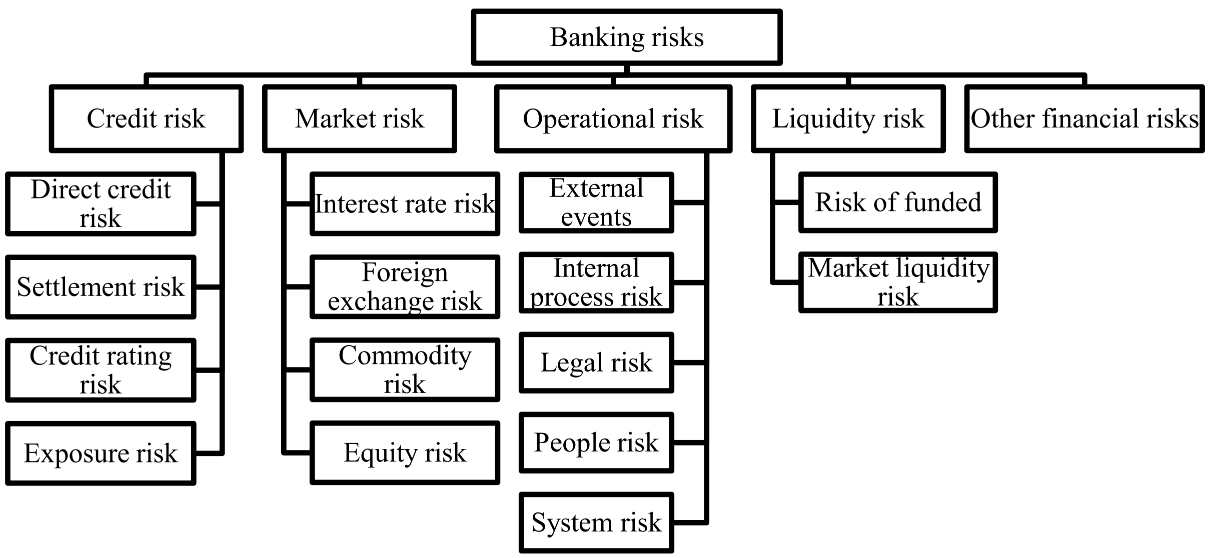
This chapter mainly focus on some financial risks which banking system meets in their daily operation such as credit risk, market risk, operational risk, liquiridty risk, and some other finanical risk.

In banking system, risks come from various sources in daily operations. For example, it can be a risk when borrowers submit payments later and fail to repay their debt. In this case, it is really significant to monitor, manage and measure these risks which considered as risk management function of a bank.

Being one of the most regulated industry, banks are known as the most stable and too-big-to-fall institutions. Regulatory capital requirement and equity capital are only one of thousands of factors to build visibility and confidence of consumers. Thanks to good risk management, banks minimize the risk to receive a good position over a long period time.

Based on Basel Accords, there are four main types of financial risks containing credit risk, market risk, operational risk and liquidity risk. On the other hand, there are some more risk as well focus on other different reason. Shown in *Fig. 2.1*, we can see the specific classification of each type of risk.

Fig. 2.1 Banking risks



More specificately, in *Tab. 2.1*, we collect data from top 4 banks in China and calculate the generated table including risk-weighted-assets and capital requirement of each risk classification by summing them up. Data is from the annual report of ICBC, CCB, BOC and ABC.

Tab. 2.1 RWA and capital requirement of top 4 chinese banks (million CNY)

	RWA	Capital requirement	%
Credit risk-weighted assets	41,819,404	3,345,552	86.44
Market risk-weighted assets	633,246	50,660	1.31
Operational risk-weighted assets	4,143,262	331,461	8.56
Additional risk-weighted assets	1,783,272	142,662	3.69
Total RWA	48,379,184	3,870,335	100

Source:Own calculation.

As a result, credit risk is the most important risk and the risk-weighted assets of credit risk accounts almost 86.44% for top 4 banks in China. It can be also explained by capital requirement and the total capital requirement for top 4 Chinese banks is 3,345,553 million CNY.

2.1 Credit risk

In this section, the credit risk would be described in details. Firstly, several types of credit risk would be introduced including exposure risk, settlement risk, credit rating risk and exposure risk. And then, there are some factors affecting credit risk such as exposure at default, loss given default, possibility of default and maturity. At last, there are some ratio indicators to measure credit risk by financial statements.

Credit risk is defined as "*the potential that a bank borrower or counterparty will fail to meet its obligations in accordance with agreed terms*" by Basel Committee on Banking Supervision (2000). Due to absorbing deposits and providing loans, banks act as middleman between borrowers and lenders. In this process, banks are able to transform little amount, short maturity and high liquidity deposits to large amount, long maturity and low liquidity loans. Thus, credit risk is the most important risk banks faced and it is significant that bank managers are able to try their best to minimize the credit losses by diversification of assets. By providing loans to different credit rating clients with different amount and maturity loans, assets can be diversified associated with lower expected return and lower credit risk.

In China, risk management in banking system should be regulated by China Banking and Insurance Regulatory Committee (CBIRC) which established in Aug. 2018 by merging China Banking Regulatory Committee (CBRC) and China Insurance Regulatory Committee (CIRC). Under the regulation and supervision of CBIRC, we have five types of loans including pass, special mention, substandard, doubtful and loss based on the possibility of collecting the principal and interest of loans. Substandard, doubtful and loss loans are supposed to be non-performing loans.

2.1.1 Types of credit risk

To better understand what credit risk is, it is necessary to define the different types of credit risk. Based on different reason, we have four categories of credit risk, which is,

- direct credit risk,
- settlement risk,
- credit rating risk,
- exposure risk.

Direct credit risk

Direct credit risk, also known as default risk, occurs when debtors cannot payback their debt either fully or partially. It is considered that events potentially qualify as default due to many reasons, such as delaying payments temporarily or indefinitely, restructuring of debt obligations by reason of bad financial condition, bankrutcies and so on. If the debtor is government, it is so called sovereign risk. For example, resulting from subprime crisis in 2008, the large Swiss bank UBS announced a 10 billion USD loss. These credit losses mainly came from high-risk clients such as some subprime mortgage borrowers and they cannot repay their loans. With the spread of subprime crisis, global economy downturn causes bankruptcy of Greek government.

Settlement risk

Settlement risk refers to a possibility that one party of contract fails to deliver or pay either underlying assets or cash value of contract to another party. It usually occurs when one party already pays the money and another party doesn’t deliver underlying assets or one party already deliver and another party doesn’t pay. In this case, settlement risk takes place over a short period. In banking system, there are two types of settlement risk, foreign exchange settlement risk and commercial paper settlement risk.

Credit rating risk

As the name implies, this risk associate with credit rating and credit rating is an evaluation of likelihood of default of debtors. For corporates, credit rating usually provides by external rating agencies such as Standard&Poor’s, Moody’s and Fitch Ratings. For individuals, by integrating debtor’s characters, financial conditions, collaterals, capacity and capital, credit quality and credit score of individual are determined. In China, Individual Credit Reporting System provides credit rating for individuals. Credit Reporting System, established by People’s Bank of China, gathers all the credit records from banks, internet finance and other lending program. When there is either a bad or a good credit rating of debtors whether by rating agencies or by internal credit rating model, banks face potential losses but one clear difference is that the potential losses are much lower if debtors have a good credit quality.

Exposure risk

Exposure risk arises from uncertainty of future exposure. Exposure for loans is the sum of number of principals and interest accrued. Bank loans can be also divided into several types. One typical bank loan is term loans which has fixed maturity and contractual repayment schedule. Exposure risk occurs due to the possibility of failing to repay the principals and renegotiating. Another typical bank loan is line of credit, which allows debtors a committed amount of credit and debtors decide when and how much they withdraw. When the total lines of credit are determined, the draw amount and undraw amount of credit lines are uncertainty since individual borrowers decide on how much to withdraw.

2.1.2 Factors affecting credit risk

It is significant for managers to measure credit risk for better position of banks. There are many related events affecting credit risk such as the financial condition of borrowers, situation of credit extension, historical credit records and so on. In general, four important factors should be taken into consideration, which is,

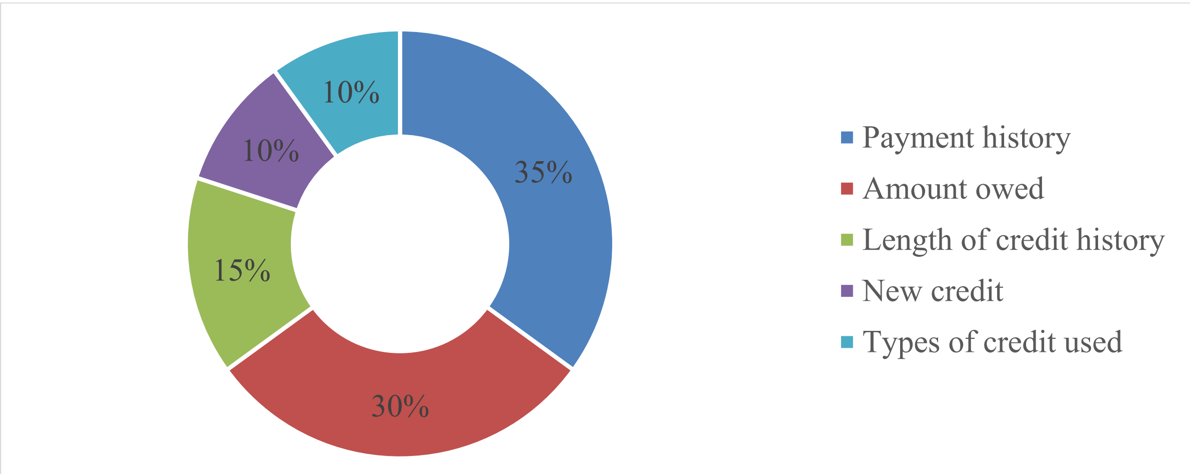
- probability of default (PD),
- exposure at default (EAD),
- loss given default (LGD),
- maturity.

Probability of default

Probability of default, also known as default probability, refers to degree of default of a borrower. In general, the higher the probability of default, banks may charge more interest for compensating higher risk, therefore, the higher interest rate. In this case, it is very important for banks to identify the degree of default. Usually, for individuals, FICO score is a significant way and for corporates it depends on credit rating. Moreover, it can be measured by historical credit data.

FICO score is a credit score estimated by Fair Isaac Corporation. It is a quantitative model to reflect credit risk. Banks can use FICO score to determine whether they would extend credit to borrowers. FICO score measure by various criteria such as historical record, payment, current level of financial conditions, length of credit history and new credit account. *Fig. 2.2* shows how much different creteria accounts for scores.

Fig. 2.2 FICO credit score factors



Source: FICO score facts sheet from FICO company.

In general, payment history and amounts owed accounts the largest part of score and they accounts 35% and 30%, respectively. The longer the length of credit history, the better of historical records, the lower degree of indebtedness, and the more numbers of credit accounts, the higher the scores. As long as we have score, clients can be devided into several level based on their credit situation. In *Tab. 2.2*, we can see different range of score representing different situation when clients applying for loans.

Tab. 2.2 Category and score range

Category	Range	Impact
Very Poor	300-579	Applications will not likely be approved for credit
Fair	580-669	Applicants may be approved for some credit, though rates may be unfavorable and with conditions such as larger down payment amounts.
Good	670-739	Applicants may be approved for credit but likely not at competitive rates
Very Good	740-799	Applicants likely to be approved for credit at competitive rates
Exceptional	800-850	Applicants most likely to receive the best rate and most favorable terms on credit accounts

Source: FICO score facts sheet from FICO company.

If the score is above 670, it means our credit is quite good and if our score is lower than 579, lenders may think it is a risky borrower and do not borrow money. Moreover, the higher the score, the easier application would be approved and also lower the interest rate and term.

For corporates, the credit rating is mainly based on credit rating agencies. Credit rating agencies assess both rating of issues and issuers, and, rating is usually based on letters, numbers, words, and even the combinations of these in each rating scale. The credit rating from S&P, Moody's and Fitch is widely used in the world, even though there is quite different with expression of rating scales in different agency. Generally, it can be summarized in *Tab. 2.3*.

Tab. 2.3 Long-term rating matrix

	S&P, Fitch	Moody's
Best quality companies	AAA	Aaa
Higher risk than AAA	AA	Aa1 Aa2 Aa3
Economic situation can effect finance	A	A1 A2 A3
Medium class which are presently okay	BBB	Baa1 Baa2 Baa3
Non-investment grade (speculative or junk bond)	BB	Ba1 Ba2 Ba3
	B	B1 B2 B3
	CCC	Caa1 Caa2 Caa3
	CC	Ca
	C	C
	D	-

Source: Rating scale of definition of each company.

Intermediate modifiers, such as a plus(+) and a minus (-), are used by Standard & Poor's as well as Fitch for each category between AA and CCC to show relative standing with the rating category. And these raing expression can be one-to-one correspondence to the detail of same rating category of Moody's, for example, the meaning of AA+ is approximately same with the meaning of Aa1. Obligations and issuers rated under BBB or Baa are regarded as having significant speculative with huge uncertainties thus they are considered as non-investment grade. Obligations rated D means in default.

Eventhough there is a very good credit rating of one corporate, credit rating is possible to be changed because the different situation of financial conditions over the year. In the CreditMetrics model, risks are considered as not only from default but also from changes in value due to upgrade or downgrade of obligors. In this case, it is important to estimate not only the probability of defaut but also the chance of migrating. There is an example illustrated in *Tab. 2.4* which is corporate one-year transition matrix of Fitch in 2016.

Tab. 2.4 Corporate finance one-year transition matrix in 2016 (%)

Credit ratings as of 31/12/2016	Credit ratings as of 31/12/2017							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	100.00	-	-	-	-	-	-	-
AA	-	94.20	0.72	-	-	-	-	-
A	-	0.88	93.44	2.90	0.38	-	-	-
BBB	-	-	1.14	90.34	3.98	0.08	0.24	-
BB	-	-	-	4.79	80.64	7.98	0.20	0.60
B	-	-	0.31	-	6.54	77.57	3.43	3.43
CCC	-	-	-	-	-	34.78	42.03	20.29

Source: Fitch: Global corporate finance 2016 transition and default study.

In this transition matrix, it shows the probability of both upgrade and downgrade of each corporate. Shown in *Tab. 2.4*, the first column from left-side is credit rating at the end of 2016 and second row is the credit rating at the end of 2017. For instance, the figure of 20.29 at the rightmost bottom shows there is 20.29% of probability of corporate rated CCC in 2016 transforming to corporate rate D in 2017 which is downgrade.

Exposure at default

Exposure at deault, EAD for short, measures the sum of total loss of banks when clients default in the future. Generally, it is complex to computate EAD because it is related with the types of product. Due to the fact that it depends on wether it is term loan or line of credit, it is generally unkown at the current date due to its randomness of size. For instance, when the amount of loans are fixed which means a term loans with fixed interest rate such as mortgage, banks are easy to computate EAD by using current ountstanding amount and contractural schedual. However, it is hard to calculate EAD if the future cash flow is stochastic. For example in the case of floating interest rate, the schedual payments are driven by market indexes for floating-rate loans and we cannot simply calculate EAD only by current information. Or the future cash flow are driven by the willings of clients such as line of credit. Specifically, there are two components of committed line of credit, drawn amount and undraw amount. The drawn amount is the cash effectively borrowed and the undrawn amount remains a part of committed line of credit. Both drwan amount and undrawn amount are unkown at current date and depend on the willing of clients' behaviour in the future.[1]

Loss given default

Loss given default is the actual loss when banks suffer from default. Usually, it is a percentage loss on exposure resulting from a default. According to Deloitte, there are two types of LGD shows totally different information, performing LGD and defaulted LGD. Performing LGD estimate the loss for defaults within one year with pre default information and defaulted LGD estimate for already defauted clients the loss with up-to-date information.

For calculation of LGD, the relationship between LGD and recovery rate can be used here. The sum of LGD and recovery rate is fixed and equals to 1 which give a good way to computate LGD. The relationship can be represented as,

$$Loss\ given\ default = 1 - Recovery\ rate. \tag{2.1}$$

[1] Source: BESSIS, J.: *Risk Management in Banking*, Chichester: John Wiley & Sons Ltd, 2003. 201 p. ISBN 0-471-89336-6.

Recovery rate represents how much money banks can get back when there is a default. It is a percentage of exposure recovered after a default. For example, mortgage usually means a recoveries of colleteral and corporate loans have multiple recoveries. In the event of a default, the recovery rate can be estimated depends on the seniority classification and different seniority class of debt has defferent recovery rate. *Tab. 2.5* below collect the recovery rates in the even of default as reported by Moody's Investors Services.

Tab. 2.5 Recovery rates by seniority class (% of face value)

Seniority Class	Mean (%)	Standard Deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.9

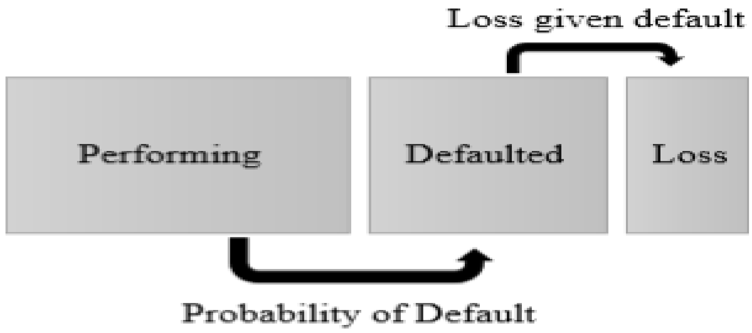
Source: Carty & Lieberman [96a] —Moody's Investors Service

In this table, we can see if there is a senior unsecured bond, the mean value of recovery rate is 51.13% of its face value and the standard deviation is 25.45%. In this case, we can see the with a senior unsecured bond, the LGD is 48.87%. The higher seniority class, the higher the expected recovery rate. For example, the mean of recovery rate of junior subordinated bond is only 17.09% which means the LGD is 82.91% and the recovery rate gradually increases because of higher rating. Based on Basel, there are several regulations on LGD. For those senior unsecured bonds on corporates, sovereigns and banks are assigned a 45% LGD and subordinated claims on corporates, sovereigns and banks are assigned a 75% LGD. The estimation of own LGD by banks can be allowed in the advanced approach.[2]

We consider exposure at default is the total potential loss when a client defaults and probability of defaut is the probability that a default occurs. The relationships between these two componets and losses can be represented as *Fig. 2.3* below.

[2] *Source: BESSIS, J.: Risk Management in Banking, Chichester: John Wiley & Sons Ltd, 2003. 203 p. ISBN 0-471-89336-6.*

Fig. 2.3 The relationships among PD, EAD, LGD and Expected loss



Source: The use of Loss Given Default from Deloitte

As *Fig. 2.3* shows, the defaulted value should be performing outstanding times probability of default and the expeted loss equals to loss given default times defaulted value, which is,

$$Expected\ loss = PD \cdot EAD \cdot LGD. \tag{2.2}$$

Maturity

Maturity of loans is the last one important factor affecting credit risk. To better regulation and accounting purpose, there are three types of loans based on different maturity, short, medium and long term loans. The maturity of short loans usually less than 1 year and the maturity of long term loans is longer than 5 years. The shorter the maturity, the better the liquidity and the lower risk.

2.1.3 Ratio indicators of credit risk

Next, some ratios to measure the credit risk are described.

NPL ratio

NPL ratio is connecting with those loans delaying repaid to banks which is called nonperforming loans, usually past due around 91 days. We use NPL ratio to measure how much percentage nonperforming loans accounts for total loans.

$$NPL\ ratio = \frac{NPL}{Total\ loans\ and\ lease}. \tag{2.3}$$

Nowadays in China, NPL ratio is a very important indicator to measure if the bank perform well. From the last decade, the average of NPL ratio in China was 1.39% and, in 2018, the NPL ratio reached at 1.89%.

Provision ratio

Banks need to make some provisions for those loans who have probability to default which we called provisions for loan loss. It usually showed in income statement and should be deducted to calculate net income. In this case, we have provision ratio (PR),

$$PR = \frac{Annual\ provision\ for\ loan\ loss}{Total\ loans\ and\ lease}. \tag{2.4}$$

Charge off ratio

After a period of time, part of NPLs proves not to be paid back due to bankruptcy of clients or other factors. Managers need to take these worthless assets away from balance sheet. In this process, we have charge-off ratio to measure how many assets is worthless in one year,

$$Charge\ off\ ratio = \frac{Net\ charge\ off\ loans}{Total\ loans\ and\ lease}. \tag{2.5}$$

Loan loss allowance ratio

The provision ratio and charge off ratio can only measure the credit risk situation in one year. For measuring total credit risk in banks, we have loan loss allowance (LLA) ratio. LLA is accumulative provisions.

$$LLA_t = \sum Provision = LLA_{t-1} + Provision_t - Charge - off_t, \tag{2.6}$$

$$LLA\ ratio = \frac{LLA}{Total\ loans\ and\ lease}. \tag{2.7}$$

Coverge ratio

Next, we have coverage ratio (CR) measuring how much nonperforming loans can be covered by loan loss allowance. The coverage ratio should be higher than the loss given default.

$$CR\ ratio = \frac{LLA}{NPL}. \tag{2.7}$$

2.2 Operational risk

The operational risk nowadays has been considered more and more important in banking system. Here, we introduce several types of operational risk and then the operational loss events are described.

2.2.1 Types of operational risk

Operatoinal risk relates to problems in internal process and also external events which affecting baking system. There are several types of operational risk based on different operational events,

- internal process risk,
- people risk,
- systems risk,
- external risk,
- legal risk.

Internal process risk

Internal proces risk is the potential losses associated with the internal process of banks. Problems occur when a process is not perfectly organized and is inefficient. For example, if the system is lack of controls, there is a failure to record transactions in accounts. Other examples include marketing error, money laundering, failure to reporting or documentation, transaction error and internal fraud.

Poeple risk

People risk is related to employee error or fraud. This is due to high employee turnover, poor management, inadequate staff training and over-reliance on key employees. In this case, employees should be more careful to avoid people risks.

System risk

System risk refers to the problem of high-tech systems such as computer systems. The day-to-day operations of banks rely on efficient computer systems. Computer system failures can be caused by a various reasons, such as data corruption, improper project control, and programming errors.

External risk

External events affect the day-to-day operations of banks as well, and, although the likelihood of these events is very rare, it has a major impact on the bank's operations. These external events include events outside which impact whole industry, external fraud and theft, terrorist attacks and transpot system interruption.

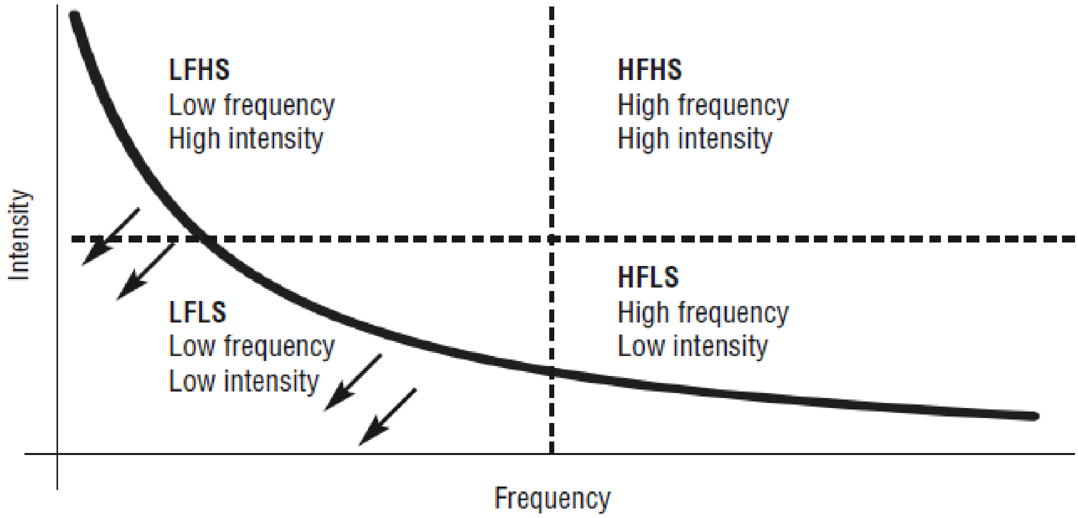
Legal risk

The uncertainty of legal actions or the interpretation of contracts, laws and regulations leads to legal risk. It is very rare but huge impact for banking industry in a region.

2.2.2 Operational loss events

There are lots of events that may result in operational loss and we cannot have a canonical list. In this case, operational loss can be divided intoe several parts by its frequency and how much the potential losses. *Fig. 2.4* list four specificately types of opearational loss events.

Fig. 2.4 Loss intensity and frequency chart of operational risk events



Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 188p.

As *Fig. 2.4* shown, managers may focus on two types of events which is high-frequency and low-intensity of losses (HFLS) events and low-frequency and high-intensity of losses (LFHS) events. It is because the cost of monitoring of events with high frequency and impact, and events with low frequency and impact is higher than the losses itself. For operational risk management, managers try their best to lower the losses of high-frequency events and the frequency of high-impact events.

2.3 Market risk

In this sub-chapter, there is the description of market risk. Firstly, several types of market risk is destinguished. Then we focus on the probable methodology of measuring market risk.

Market risk is asscociated with the daily fluctuation of financial markets. Generally, market risk can divided into two parts, systematic risk and specific risk. Systematic risk affects the market price of similar financial assets or whole financial markets and specific risk suffers the fluctuation of the price of an individual asset resulting from daily operations.

2.3.1 Types of market risk

There are four types of market risk by different factors affecting market risk,

- interest rate risk,
- foreign exchange risk,
- commodity risk,
- equity risk.

Interest rate risk

Interest rate risk is one of market risk associated with interest rate change. In this case, it mainly affects interest-rate-sensitivity assets and liabilities rather than fixed-reate assets and liabilities. For example, if interest rate increase, the value of long-term assets would tend to fall more than the value of short-term liabilities. Furthermore, if interest rate rise, change of the income of long-term assets such as loans would lower than the change of the expense of short-term liabilities such as deposts resulting in the decrease of bank's equity.

Moreover, interest rate risk can be explained by formula listed below,

$$Interest\ rate\ risk\ ratio = \frac{Interest - sensitive\ assets}{Interest - sensitive\ liabilities}. \tag{2.8}$$

Foreign exchange risk

Foreign exchnage fluctuation affects the increase or decrease of bank's equity as well. Beside taking deposits and receiving loans, another function of banks is to buy and sell foreign exchange on behalf of clients those who need to pay their international transactions. The exchange rate related activities would be affected by exchange rate fluctuation and there is much uncertainty in foreign exchange market. In this situation, it is really important for banks to hedge this foreign exchange risk.

Commodity risk

Commodity risk asssociated with the adverse change of commmodity price. The changes in demand and supply in the market affect the value of commodities. There are several types of commodities taken into account such as algricultrual commodities, industrial commodities and energy commodities.

Equity risk

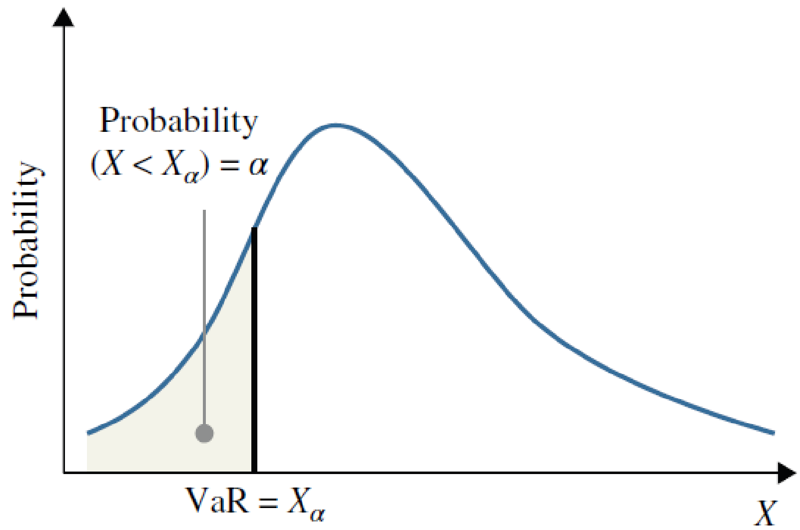
Equity risk refers to a potential risk of a fluctuation of equities such as stocks and shares. Banks suffer equity risk resulting from purchasing the ownerships of other companies. In this case, managers should use proper portfolio to reduce risk.

2.3.2 Value-at-risk

Nowadays, Value-at-risk is the most common way to measure maket risk. Value-at-risk, also abbreviated as VaR, is the maximum loss in our portfolio with a given confidence interval. Different confidence interval reflects different risk attitude, for instance, the confidence interval of Citibank is 94.5% and for banks in European is 99% according to Basel regulation.

VaR method measure the potential loss in the future, usually next day, week or year. In this case, VaR is defined as a quantile of the distribution of the variations of value and the distribution can be normal distribution or not normal distribution. *Fig. 2.5* as below shows the graphical interpretation of VaR.

Fig. 2.5 Graphical interpretation of VaR



Source: BESSIS, J.: Risk Management in Banking, Chichester: John Wiley & Sons Ltd, 2003. ISBN 0-471-89336-6. 124p.

Illustrated in *Fig. 2.5*, with a given comfidence level α, VaR equals to the α-quantile of distribution which is X_α. It means the maximum potential loss in the future managers can affort. In this case, VaR should obey the following relationship,

$$P(\Delta V > X_\alpha) = 1 - \alpha \;\; or \;\; P(\Delta V \leq X_\alpha) = \alpha. \tag{2.9}$$

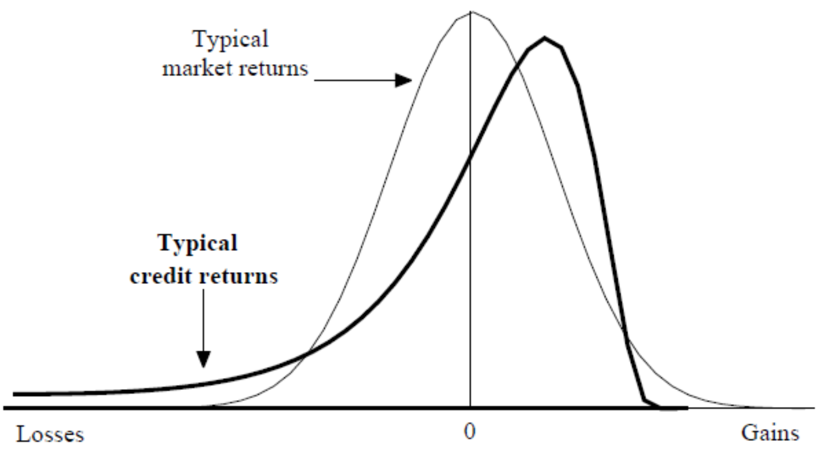
Where ΔV measures the changes of the value of assets.

Usually, the left-side tail of distribution shows the loss of portfolios and right-side tail shows the gains of portfolios.

2.3.3 Difference between credit risk and market risk

VaR is the most common tool to measure the market risk as we discussed before, however, there is no probability to measure credit risk by totally same method. The biggest problem is due to the fundamental difference between the distribution of credit return and market return. The equity returns are relatively symmetric and follow normal distribution. In this case, the α-quantile reflecting VaR of market risk can be estimated by mean value and standard deviation of portfolio value. However, actual credit returns are skewed and with fatter tails compared to normal distribution which means losses are more frequent than gains. In this situation, it is insufficient to estimate quantile by mean value and standard deviation and we need more statistical data. *Fig. 2.6* represents the comparison of distribution of credit returns and market returns.

Fig. 2.6 Loss intensity and frequency chart of operational risk events



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 7p.

Illustrated in *Fig. 2.6*, the mean value of market return is lower than the mean value of credit returns. And the left-side tail of credit returns is higher than the left side tail of market returns due to default of credit. In this case, with the same confidence level, the potential loss of credit returns is higher than the potential loss of market returns. Managers usually use 95% confidence level for market risk, however, for credit risk, 99% confidence level is more preferred.

2.4 Liquidity risk

Next, there is an introduce of liquidity risk in banking system. In this sub-chapter, types of liquidity risk would be described and then there are some ratio indicators for liquidity risk.

2.4.1 Types of liquidity risk

Liquidity risk is the risk associated with the shortage of cash and cash equivalent assets and banks have potential inability to meet its payment obligations. Liquidity refers to the ability to convert assets into cash quickly with a little or no loss. There are three types of liquidity risk, which is,

- risk of changing refinancing interest rate,
- risk of insolvency,
- marekt liquidity risk.

Risk of changing refinancing interest rate and risk of insolvency relate to the shortage of funded. In this case, banks have insufficient money to pay back to their clients. For example, people prefers to withdraw their money when there is a financial crisis. The outflows of money is higher than the inflows resulting in situation that not all clients can get back their money. This is risk of insolvency. Banks may change their business model to cover the outflows, for example, borrowing money from other banks when maturity of loans is longer than the maturity of deposits. However, the interest rate of borrowing may higher than the original interest rate which is risk of changing refinancing interest rate. Market liquidity risk relates to the liquidity of market. When there is very low liquidity of market, the price of instruments would be lower than its original value.

2.4.2 Ratio indicators of liquidity risk

Financial statement plays an important role in measuring liquidity risk. The most common ratio used in banking system is liquidity coverage ratio (LCR) and net stable funding ratio (NSFR).

LCR

LCR reflects the minimum level of liquid assets when market shocks and the liquidation value of shor-term assets should be higher or equal to the unexpected outflows over a given period. Unexpected outflows may due to a significant downgrade of the institution's public credit rating, a partial loss of deposits and so on. It can be defined as,

$$LCR = \frac{Stock\ of\ high - liquidity\ liquid\ assets}{Total\ net\ cash\ outflows\ over\ the\ next\ 30\ days}. \tag{2.10}$$

In this case, LCR should be higher or equal than 1. Note that the high-liquidity liquid assets contains not only government and public sector entity assets, but also the high-rating corporate bonds.

NSFR ratio

Net stable funding ratio aims to provide stable sources of financing fund over a longer time horizon than LCR ratio. Time horizon of NSFR is usually one year and, during this period, it is imposed that the resources that banks need should be at least higher than the assets they have. NSFR is defined as the ratio of available stable fund (ASF) to the required stable fund (RSF), which is,

$$NSFR = \frac{Available\ stable\ funds}{Required\ stable\ funds}. \tag{2.11}$$

3 Description of the credit risk management and models

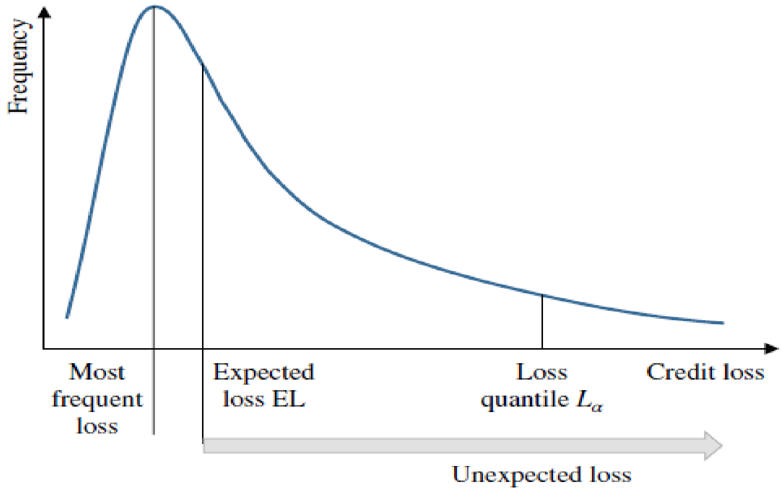
In Chapter 3, there is a description of several methods of credit risk management including scoring models, rating systems and portfolio models. After that, the main part is the description of CreditMetrics model to make clear the process of measuring credit risk. At last, we focus on how to calculate capital requirements and also regulation of capital requirement based on Basel Committee.

3.1 Difference between expected loss and unexpected loss

Before the discussion of credit rating modeling, it is useful to distinguish the expected loss and unexpected loss. In this subchapter, expected loss and unexpected loss are introduced. And then, the calculation of economic capital is described following behind.

As we mentioned in Chapter 2, the expected loss is drived by three component which is PD, LGD and EAD as shown in *Equation (2.2)*. It is usually considered as a mean value of the probability distribution of future losses. Lenders estimate the expected loss as ex-ante loss in practice and charge a certain amount of interest to hedge its risk. As for unexpected loss, it is the reason why the real loss has bias with expected loss. It is defined as the variability of the loss around its mean value. As same as market VaR, we can estimate maximum unexpected loss by analyzing the distribution of loss with certain confidence level. The distribution is highly skewed from left with small portfolio losses being the most frequent due to diversification. The loss distribution is shown as follow *Fig. 3.1*.

Fig. 3.1 Loss distribution of credit risk



Source: BESSIS, J.: Risk Management in Banking, Chichester: John Wiley & Sons Ltd, 2003. ISBN 0-471-89336-6. 209p.

As shown in *Fig. 3.1*, we can see that the distribution skewed to right and the expected loss is higher than the most frequent loss. The unexpected loss can be found on the right-side tail which is above the expected loss. With α confidence level, we have the α-quantile of loss distribution which is L_α. This α confidence level represents the default posibility of banks and therefore influence its posibility of insolvency and its ratings. Hence, an appropriate confidence level should be very low and it is well below 1%. Moreover, we can also calculate economic capital. *"The economic capital can be seen as a buffer against the unexpected loss in excess of the expected loss„*[3]. And it can be expressed as follow,

$$K_\alpha = L_\alpha - EL. \tag{3.1}$$

Where K_α represents the economic capital with α confidence level, EL means the expected loss.

Economic capital reflects the capital level a bank must maintain to cover the large but unexpected loss for survival in the long period of time. In our case, economic capital is the difference bwteen the α-quantile of loss distribution and expected loss.

[3] Source: BESSIS, J.: Risk Management in Banking, Chichester: John Wiley & Sons Ltd, 2003. ISBN 0-471-89336-6. 210p

It is signifiacant to make clear the difference between the expected loss and unexpected loss when dealing with a diversified portfplio. Although the total expected loss of porfolio can be simply defined as a sum of the expected loss on each loans, the total portfolio loss take unexpected loss into account because its uncertainty. However, the volitilities of total portfolio is usually lower than the sum of the volatilities of the losses on each loans. This is because that we can reduce unexpected loss by decreasing the correlation between each loans, in other words, diversifying portfolios. It means that diversifying portfolios with a certain expected return can significantly reduce total credit risk.

3.2 Models of credit risk managemnt

In this section, several models of measure credit risk management are introduced including credit-scoring model, raiting system and portfolio models. These credit risk model give possibilities to estimate expected loss.

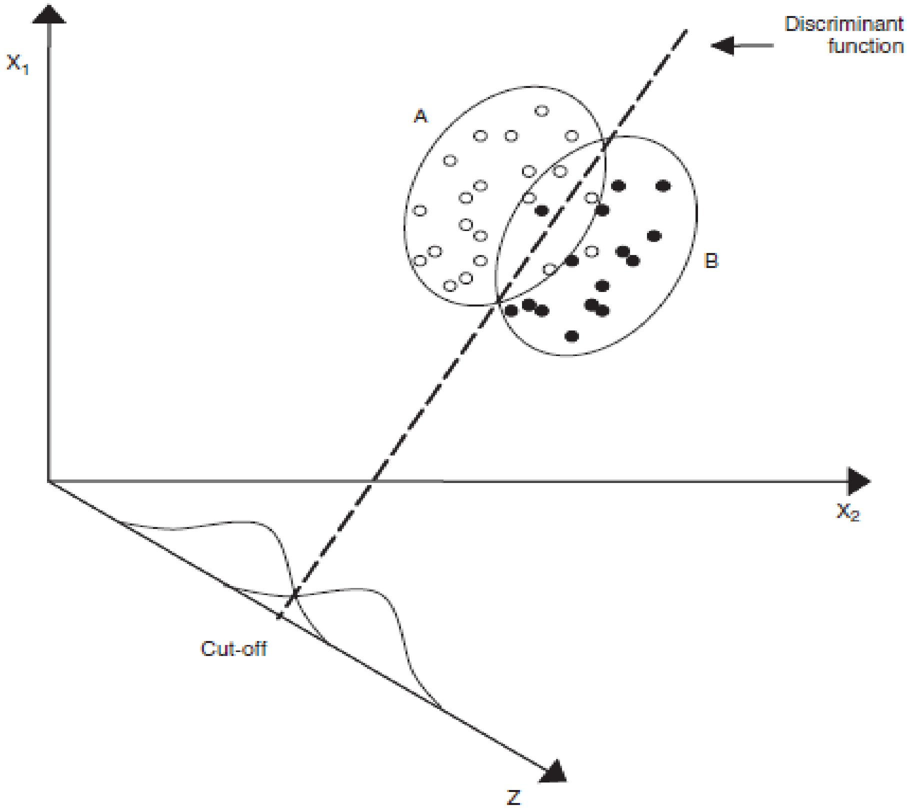
3.2.1 Credit-scoring Models

Credit-scoring models are the most common use of statistical models to forecast a company's default. It was first established in 1930s by Fisher (1936) and Durand (1941) and then the development and spread of these models were in 1960s thanks to Beaver (1967) and Altman (1968) and other authors. The credit-scoring model is a multivariate model including the main economic and financial indicators of a company, each of them has its own weight to influence the finall results, to forecast default. After the analysis, the final results can be used to assess the borrowers' probability of default by an index of creditworthiness with a numerical score.

In our thesis, we mainly focus on linear discriminant analysis, the studied by Fisher as early as 1936, and its development studies by Edward Altman in 1968.

Linear discriminant analysis is based on a deductive approach and it mainly analyze the probility of default of a company by its economic and financial data from fiancial statement. In this case, the independent variables are usually some ratios which are easy to observed, such as accounting ratios. *"Basically, discriminant analysis is a classification technique which uses data obtained from a sample of companies to draw a boundary that separates the group of* *reliable ones from the group of insolvent ones„*[4]. The distinguish of defaulters and non-defaulters is based on the discriminant function. *Fig. 3.2* shows the Fisher model in the simplicated case whose aim is to figure out which one is more reliable (A) and another one is insolvent company (B) by analyzing the two variables, x_1 and x_2.

Fig. 3.2 Graphical representation of linear discriminant analysis



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 288p

Illustrated in *Fig. 3.2*, we can see that x_1 and x_2 are independent variables and the credit scores are shown in z-axis. In general, z-score with n independent variables x_j can be calculated as,

[4] ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 287p.

$$z = \sum_{j=1}^{n} \gamma_j x_j \,. \tag{3.1}$$

According to *Equation (3.1)*, z-score is defined as the combination of all indepent variables and we have the mean value of score of group of healthy companies and abnormal companies. There is also a way to calculate the specific score of every company in the group and, for ith company, the score will be computed as follows,

$$z_i = \sum_{j=1}^{n} \gamma_j x_{j,i} \,. \tag{3.2}$$

Note that coefficients γ_j are chosen to estimate z-score to better discriminat healthy and abnormal companies.

There are many studies that explain the scoring system and provide numerical results. As an example, the Altman z-score model for corporations is well known. In this Altman z-score, several accounting ratios and the coefficients γ_j are selected. It can be used for not only large corporaters but also small to medium size firms.

There are two catogories of Altman z-score, z-score for manufaturers and z"-score for non-manufactuers. The defference between these two scores are selected ratio and its weights. z-score for manufaturers can be computed as follows,

$$z_i = 1.2 \cdot x_{i,1} + 1.4 \cdot x_{i,2} + 3.3 \cdot x_{i,3} + 0.6 \cdot x_{i,4} + 1.0 \cdot x_{i,5}. \tag{3.3}$$

Where x_1 is working capital/total assets, x_2 is retained profits/total assets, x_3 is earnings before interest and tax/total assets, x_4 is market value of equity/book value of total liabilities and x_5 is turnover/total assets.

The greater the score of a company, the better its financial situation and the lower their probability of default. Moreover, Altman sets a cut-off point of z-score for recognize if the company is good or not. This cut-off value was obtianed as the average between the mean value of z-score for a sample of healthy companies and the mean value of z-score for a sample of abnormal companies and the value is 1.81. If z-score of a company is higher than 1.81, it means that the financial situation is quite good on the basis of Altman's model.

In addition, z"-score for non-manufactuers is as follow,

$$z_i = 6.5 \cdot x_{i,1} + 3.26 \cdot x_{i,2} + 6.72 \cdot x_{i,3} + 1.05 \cdot x_{i,4}. \tag{3.4}$$

Comparing to z-score for manufaturers, the asset turnover ratio which is x_5 is completely removed from the model and coefficient for weights x_1, x_2, x_3 and x_4 are totally changed. Furthermore, the interpretation of score is also changed. A non-manufacture company is considered as healthy when its z"-score is higher than 2.9. The conpany should be caution when the z"-score is between 1.23 and 2.9. The probability of default will be considered as high according to Altman model when z"-score is lower than 1.23[5].

3.2.2 Rating system

The discriminant analysis is a quantitative method to estimate the creditworthness of a borrower. However, there ase some qualitative approches can be used to assess the credit risk. *The qualitative models are based on non-automatic evaluations carried out by human experts analyzing company data*[6]. These qualitative models nowadays are widely used by international rating agencies such as Moody's, Standard & Poor's and FithRating. In recent years, the credit rating in banking systems are assessed with the help of both quantative and qualitative methods. There are three steps of rating process,

- rating assignment,
- rating quantification,
- rating validation.

Rating assignment

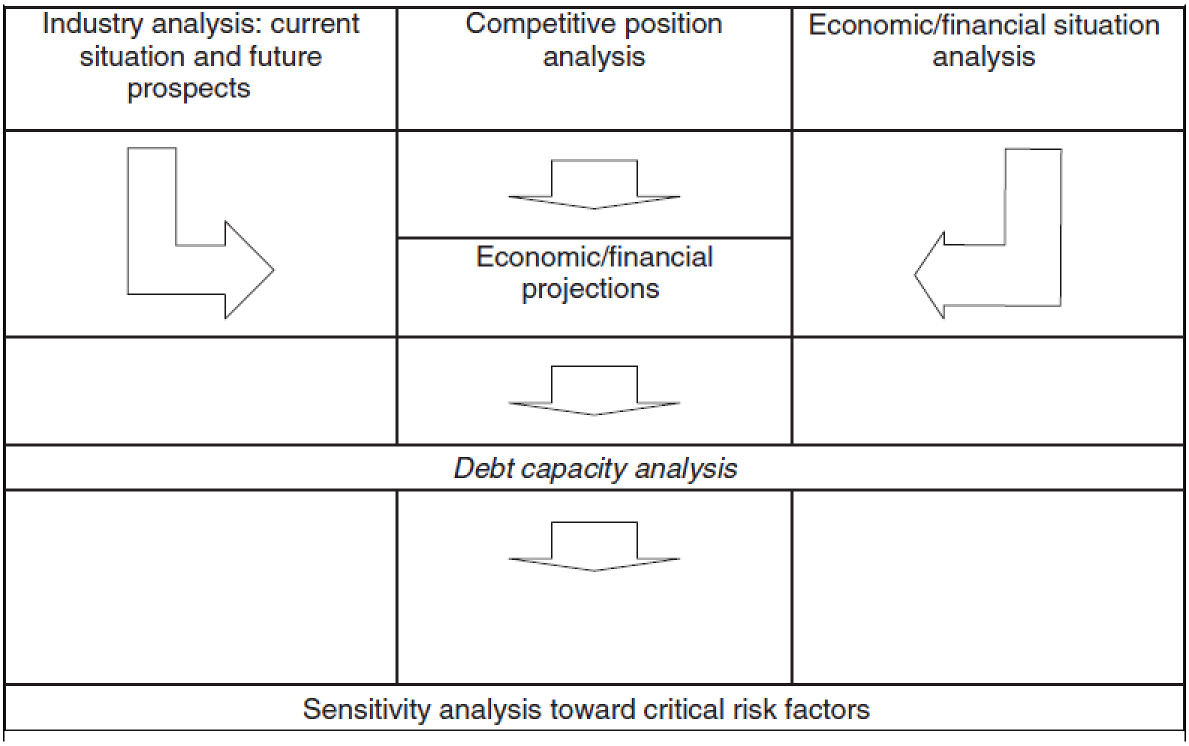
There are two types of credit rating based on different rating agencies, credit rating by rating agencies anf internal credit raing by banks. They are both advantages and disadvantages. For instance, rating by agencies may avoid problem of asymmetric information and internal

[5] Source: http://www.creditguru.com/index.php/bankruptcy-and-insolvency/altman-z-score-insolvency-predictor-for-non-manufacturers-emerging-markets

[6] ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 369p.

credit rating may have more financial information. In either case, the assignment of rating is to give a rating grade representing an indirect estimation of its probability of default. *Fig. 3.3* shows the process of analysis underlying an agency's rating assignment.

Fig. 3.3 The process of analysis underlying an agency's rating assignment



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 375p.

Fig. 3.3 summarizes the main stages in this process. The future economic projection of a company will be drawn up after analyzing the industrial situation, financial situation of own company and also the competitive position. And then, the debt capacity can be predicted with the help of the projection by evaluating the future cash flow. At last, the sensitivity will be analyzed to assess if the company perform well in the worst case, such as reduced demand, reduced efficiency, rising interest rates or other adverse events.

Risk quantification

Companies will be assigned a latter after rating assignment and there is a problem how to convert the latter-rating into a quantitative element such as probability of default. There are three possible approaches to this problem,

- the statistical approach,
- the actuarial approach,
- the mapping approach.

The statistical approach is based on the credit-scoring models to assess the probability of default. It is quick and convinient but there are two weaknesses. Firstly, this approach depends on quantitative model and it is hard to have results when there is only qualitative valuation performed by experts. Secondly, credit-scoring models are followed by several unrealistic assumptions. As an example of descriminant analysis, it assumes that the distribution of the input variables is normal.

The actuarial model is based on the historical records of default rates. The historical default rates can be used as an reference of future probability of default of borrowers in each classification. For instance, if the records show that 1% of borrowers assigned to class BB trend to default in one year, a probability of default of 1% will be assigned to all borrowers which rated BB in the future.

The mapping approach is a result of combination of internal rating and rating by rating agencies. Some banks may establish some relationships between the internal rating and rating by rating agencies. For instance, a 10 score out of 10 from internal rating is equivalent to Standard & Poors's AAA class.

Rating validation

A rating system should be checked if there is any change in financial condition of a company, in a word, it should be cheked periodically to maintain its effectiveness. Here are some qulitative criterias to assess the adequacy of a rating system:

- the lower the rating class, the higher the defaut rate,
- the volatility of default rate is stable over time,
- the percentage of exposures that remain in the same rating class from one year to the next year should be sufficiently high,
- migration rates toward nearby rating classes should be higher than those toward more distant class,

- most defaulting borrowers should have been classified in a low rating class for some years before the defaut took place.[7]

3.2.3 Portflio models

Credit-scoring model and rating system, as we mentioned above, focus on how to calculate probability of default of a borrower. Probability of default is a necessary parameter to compute expected loss. However, there should be some more parameters and models to calculate unexpected loss. As we discussed in *Chapter 3.1*, expected loss can be covered by amount of reserves and by interest charged to a borrower. On the other hand, unexpected loss is only covered by an adequate amount of equity. In this case, it can be regarded as economic capital absorbed by a credit exposure. The simplest way to measure unexpected loss is with the help of the standard deviation of probability distribution of future losses as same as calculating VaR of market risk. In practice, there are four models contributing to estimate unexpected losses,

- PortfolioManager™ (1993),
- CreditRisk+™ (1997),
- CreditPortfolioView™ (1997),
- CreditMetrics™ (1997).

PortfolioManager™

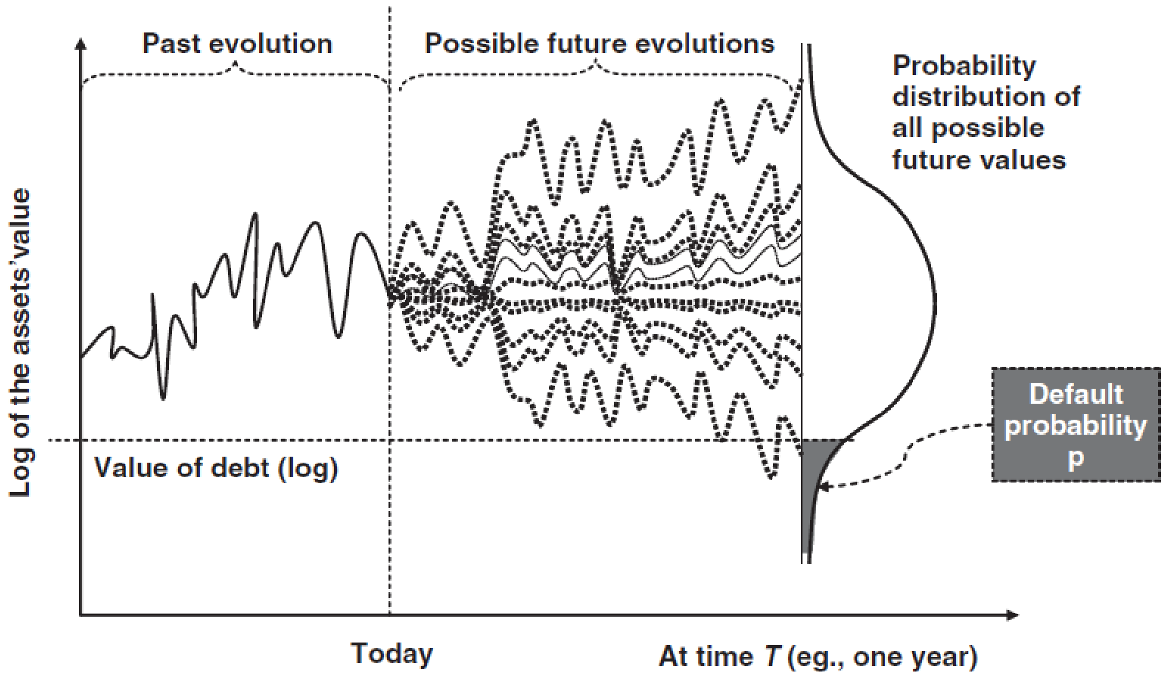
PortfolioManager™ is a model developed by company KMV, based on Merton model and the aim is to determine default porbability. Merton model give a possible way to determine the equillibrium bond spread by using stock price as input and estimate the default probability as well.

Merton model provide a structrual relationship between the default risk and the asset of a company. He assumed that the obligation of company is to repay their debt and the debt should

[7] ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 388p.

be repaid at a lump sum of principal on maturity date like zero-coupon bond. *Fig. 3.4* represents the logic behind Merton model.

Fig. 3.4 The logic behind Merton model



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 323p.

The model predict future market value of company with given face value of debt (F), time maturity (T), market value of debt (B). The difference between market value of company and debt is referred as value of risk capital. Practically speaking, the volatility of future assets' value is stochastically. It might be a reason why the longer the time horizon, the higher the risk. If the future value of assets' value is lower than the value debt, company will be insovency in the future.

The KMV model was developed by a California-based firm acquired by Moody's Investor Services, recently, to soolve the problem of estimating makret value of company at beginning, V_0, and the standard deviation of asset yield representing the volatility of the company's asset yield, σ_v. The model claims that *the value of equity (E) is equal to the value of a call option on the market value of the company's assets, with a maturity equal to the* *residual life of its debt (T) and a strike price equal to the nominal repayment face value of debt (F)*[8]. *Tab. 3.1* below shows how it works in KMV model.

Tab. 3.1 Matrix of payoff as a shareholder or for a purchase of a call option on asset value with a strike price of F

	Payoff at time 0	Payoff at T	
		if $V_T < F$	If $V_T > F$
Shareholder	$-E_0$	0	$(V_T - F)$
Purchase of a call option	$-C_0$	0	$(V_T - F)$

Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 332p.

As we can see in *Tab. 3.1,* the payoff matrix of purchasing of a call option is almost the same with the payoff matrix as a shareholder. If the market value of company is lower than value of debt now, the company face default and there is no profit but whole loss of investment, E_0. However, if the future value of company is higher than value of debt, the profit is the difference between value of company and value of debt, $V_T - F$. After calculating V_0 and σ_v, there are two additional steps to calculate default probability rather than calculate default probability, directly. Firstly, we should calculate a index of risk which associated with default risk, distance to default (DD). It can be computed as follow,

$$DP = STD + \frac{1}{2}LTD, \tag{3.5}$$

$$DD = \frac{V_0 - DP}{V_0 \cdot \sigma_V}. \tag{3.6}$$

Where DP is default point, STD means all amount of short-term debt and LTD means long-term debt.

As above formula shown, the distance to default is equal to the defference between asset value and default point, expressed as a multiple of the standard deviation of assets. However, we cannot get default probability directly from the number of distance to default. Thus, for the second step, we need to convert it into a probability of default on an empirical law which is

[8] ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 332p.

based on historical evidence. For instance, a DD of 2 could be assigned a PD of about 3% according to the past data.

CreditRisk+™

CreditRisk+™ is a simple actuarial model, developed by Credit Suisse Financial Product in 1997. This model focus on the chance of default based on the exogenous Poisson distribution of loss. It applies to credit risk due to the fact that process of estimation is similar to the process of the mathematics of insurance. Thus, the two necessary parameters of measure credit risk, similar to measurement of insurance loss, are the frequency of default events and the rate of loss given default. This model is common used in credit risk assessment of loans.

Model give possibility to calculate the probability distribution of numbers of defaults happens with a given period of time based on Possion distribution. The probability $p(n)$ with n defaults can be expressed as follows,

$$\mu = \sum PD_j , \tag{3.7}$$

$$p(n) = \frac{e^{-\mu}\mu^n}{n!}. \tag{3.8}$$

Where μ is the expeccted number of defaults, repersenting the summarize of j clients' PDs in the portfolio.

As an example of 4 defaults in the portfolio, along with 400 clients assigned 1% PD. The value of the expected number of defauts, μ, is 4. According to *Equation (3.8)*, we can compute probability $p(4)$ as follow,

$$p(4) = \frac{e^{-4}4^4}{4!} = 19.54\%$$

CreditPortfolioView™

CreditPortfolioView™, developed in 1997 by Tom Wilson, is based on economic cycle. It claims that the upgrades of migration tend to be more frequent during economic growth, moreover, the dowgrades of migration is less frequent and defauls decline. During economic reccession, it will be the opposite situation. In this case, the transition matrix should be modified based on the different phase of economic cycle according to some macroeconomics indicators such as interest rate level, employment rate, real GDP growth and so on.

In this model, the default probability can be calculated after the adjustment of macroeconomic variables based on a logit function. The default probability[9], p_{jt}, at time t or j of companies in same industry or same geographical area affecting economic cyle, can be estimated as follow,

$$P_{jt} = \frac{1}{1 + e^{-y_{j,t}}}. \tag{3.9}$$

Where $y_{j,t}$ represents the total value at time t of a health index of the j of companies adjusted by macroeconomic factors. It is a linear combination of macroeconomic variable.

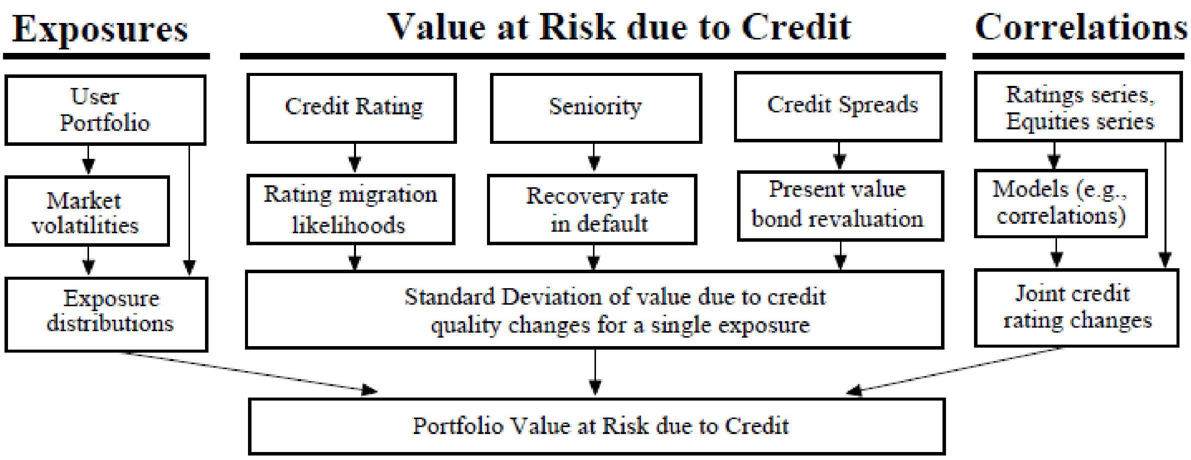
3.3 CreditMetrics™

In this section, CreditMetrics model is introduced stepy by step. Firstly, there is the process for single credit risk calculation. Then, credit qulity correlation is considered to calculate credit risk of portfolio. Thirdly, different types of credit exposure are discussed. All the descriptions in this sub-chapter are based on the Technical Document by J.P.Morgan in 1997.

CreditMetrics™ model is a futher version of RiskMetrics™, developed by J.P.Mogan originally in 1997. RiskMetrics model is to calculate market risk by means of observation of daily liquid pricing data and CreditMetrics model pays more attention to what it cannot directly observed, such as the volatility of value due to credit quality changes. The significantly defference of CreditMetrics is that the models do not assume that there is a normal distribution of return as we assumed in RiskMetrics. *Fig. 3.5* below shows the total framework of CreditMetrics.

[9] ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 426p.

Fig. 3.5 Framework of CreditMetrics



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J.P.Morgan, 1997. 4p.

The task of CreditMetrics model is to measure the portfolio VaR due to credit with the help of migration analysis. Migration analysis is the study of changes in rating class through time horizon by means of transition matrices. In this case, the model is going to find the whole range of credit quality migration which is upgrades and downgrates and historical data from a long period of time rather than the recent market fluctuations and just default. Next, the process of CreditMetrics model is discussed step by step in the following sections.

3.3.1 Single credit risk calculation

In this section, we start with the process of evaluating VaR due to credit with only one obligor. There are three steps calculating single credit exposure and it can be shown as the middle part of framework in *Fig. 3.5*. Three steps shows as follow,

- step 1: Credit rating migration,
- step 2: Valuation,
- step 3: Credit risk estimation.

Step 1: Credit rating migration

As we have mentioned above, risk comes not only from default but also from the migration in our model. In this case, credit rating migration is as same important as credit rating.

The description of migration can be found in *Tab. 2.4*. The main objective of step 1 is to determine the likelihoods of migration to any possible credit class at the risk horizon.

Step 2: Valuation

In step 2, the main task is to measure the values at the risk horizon for these credit quality states. There are two catogories of valuation, valuation of default and valuation of migration, differed by migrationm situation. If the bond downgrades to deault, it applies to valuation of default and the recovery rate should be calculated. If the credit quality migrates to another rating class rather than to default, the valuation of migration should be followed.

For valuation in the state of default, we estimate recovery rate based on its seniority class which we can see in *Tab. 2.5*. Recall the example, the mean value of recovery rate of senior secured BBB-rated bond is 53.80%, and the standard deviation is 26.86%.

For valuation of migration, the change in credit spread resulting from the migration will be estimated. In this case, we need to calculate the value of credit in one year. There are two steps in this revaluation. Firstly, we need to obtain the discount rate that can be used to discount. Secondly, we calculate present value in one year after obtaining discount rate. The discount rate is obviously not based on current rates but reflecting the possible values of market rates in one year horizon. Thus, we can use the forword rates for discount. For instance, discount rate can be one-year zero coupon forward rate. If we want to know the value a year from now, the time of first payment of coupon should be regarded as the first year and the first payment should not be discounted. Thus, the present value (PV) of credit in one year can be expreessed as,

$$PV = C + \frac{C}{(1+d)} + \frac{C}{(1+d)^2} + \cdots + \frac{C+F}{(1+d)^n}. \tag{3.10}$$

Where C represents coupon payment, d means discount rate, F is face value of bond, and n represents time to maturity.

The discount rate can be obtained from the forward curve for each rating class. There are two steps to calculate discount rate. Firstly, we need to find out the probabilities of migration to each rating category with initial defaul. It can be achieved by power the annual transition matrix of desired exponent. In this case, if it is necessary to get n-year transition matrix, the initial annual transition matrix need to multiple itself n-times. However, discount rate can be drived not only from the the transition probabilities of default rating to other rating classes but also from the forward rate. The forward rate can be computed as:

$$f_t = \frac{(1 + r_t)^t}{(1 + r_{t-1})^{t-1}} - 1, \tag{3.11}$$

Where f_t represents the t-year forward rate and r_t represents the spot rate of risk-free assets such as PRIBOR, LIBOR, EURIBOR, 2W REPO values or interest rate swap (IRS).

The relationships between forward rate and one-year discount rate can be expressed as:

$$\left(1 + r_1^d\right) \cdot \left(1 - p_1^i\right) + p_1^i \cdot RR = 1 + f_1, \tag{3.12}$$

Where r_1^d represents the disount rate of a company with assigned rating in one year, p_1^d is the probability of default in one year, RR is the recovery rate and f_1 is one-year forward rate.

The formula can be extended into two years, representing the relationship between two-year forward rate and two-year discount rate, which is:

$$\left(1 + r_2^d\right)^2 \cdot \left(1 - p_2^d\right) + \left(p_2^d - p_1^d\right) \cdot RR + p_1^d \cdot RR \cdot \frac{(1 + f_2)^2}{1 + f_1} = (1 + f_2)^2, \tag{3.13}$$

Thus, the two-year discount rate can be expressed as:

$$r_2^d = \sqrt{\frac{(1 + f_2)^2 - p_1^d \cdot RR \cdot \frac{(1 + f_2)^2}{1 + f_1} - \left(p_2^d - p_1^d\right) \cdot RR}{\left(1 - p_2^d\right)}} - 1, \tag{3.14}$$

We extend it to calculation of n-year discount rate, it can be:

$$r_n^d = (1 + f_n)\sqrt[n]{\frac{1 - RR \sum_{j=1}^{n} \frac{p_j^d - p_{j-1}^d}{1 + f_j}}{\left(1 - p_n^d\right)}} - 1 \tag{3.15}$$

Let assume a BBB-rated bond with 6% coupon rate, the face value is €100 and maturity is 5 years later. Therefore, the annual coupon payment is *€6*. *Tab. 3.2* below is an example of forward zero coupon rate for each year.

Tab. 3.2 Example one-year forward zero curves by credit rating category (%)

Category	**Year 1**	**Year 2**	**Year 3**	**Year 4**
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 27p.

Accordint to *Tab. 3.2*, we can calculate the present vaue of a BBB-rated bond if there is no migration a year from now:

$$\mu_{1,BBB} = 6 + \frac{6}{(1+4.10\%)} + \frac{6}{(1+4.67\%)^2} + \frac{6}{(1+5.25\%)^3} + \frac{6+100}{(1+5.63\%)^4} = 107.53,$$

If the issuer downgades to B one year after, the present value one year after will be:

$$\mu_{1,B} = 6 + \frac{6}{(1+6.05\%)} + \frac{6}{(1+7.02\%)^2} + \frac{6}{(1+8.03\%)^3} + \frac{6+100}{(1+8.52\%)^4} = 98.09.$$

The difference between one year after present value of BBB and B class is 9.44, which is the change of value of credit due to migration. However, there is an uncertainty that the bond will be in which rating class in the future. The eight scenarios can be shown in *Tab. 3.3*.

Tab. 3.3 Distribution of one-year market values of a BBB bond

State at year-end	Present value in one year, μ_j	Probability of migration, p_j (%)	Change, $\Delta V_j = \mu_j - \mu_{Total}$
AAA	109.35	0.02	2.28
AA	109.17	0.33	2.10
A	108.64	5.95	1.57
BBB	107.53	86.93	0.46
BB	102.01	5.30	-5.07
B	98.09	1.17	-8.99
CCC	83.63	0.12	-23.45
Default	53.80	0.18	-53.27
Mean, μ_{Total}		107.07	

Source: Own calculation.

The probability of migration is from trasition matrix. The expected value of present value in one year is €107.07 which is weighted average of each 8 scenario. If the obligor remains in its initial rating class, the one year after present value is €107.53. There is a difference between expected value and present value in initial rating class which is around 0.46. It can be regarded as a estimation of the expected loss on the bond.

Step 3: Credit risk estimation

The credit risk of portfolio, just like other risk of assets, can be represented as the standard deviation of portfolio. Hence, the main task here is to measure the standard deviation of the present value in a year. According to *Tab. 3.3*, we can easily calculate the standard deviation of all these 8 scenarios, however, we can see that the default situation is included in *Tab. 3.3*. As we have mentioned, there is an uncertainty associated with recovery rate when the bond downgrades to default. It is a problem to help standard deviation of scenario to link up the standard deviation associated with recovery rate. Here in CreditMetrics model, we have a possible standard deviation calculation in a manner that will help us to incorporate recovery rate uncertainty. The calcilation of standard deviation of values of portdolio within each state can be as follow:

$$\mu_{Total} = \sum_{i=1}^{n} p_i \mu_i \,, \tag{3.16}$$

$$\sigma_{Total} = \sqrt{\sum_{i=1}^{n} p_i \mu_i^2 - \mu_{Total}^2} \,, \tag{3.17}$$

Where p_i means probability of being in any state, μ_i means the present value a year from now within each state and μ_{Total} means the weighted average value of portfolio.

The estimation of standard deviation which incorporated by uncertainty associated with recovery rate is expressed as:

$$\sigma_{Adjusted-total} = \sqrt{\sum_{i=1}^{n} p_i (\mu_i^2 + \sigma_i^2) - \mu_{Total}^2} \,. \tag{3.18}$$

Note that the expected value of portfolio is the same as before. The only difference is the calculation of standard deviation, where we add a componet σ_i, representing the risk of each rating class and it equals to 0 when the bond do not downgrades to default. The number of σ_i when i equals to 8 can be found in *Tab. 2.5* according to seniority class.

The final result of our stand-alone portfolio with a senior secured BBB-rated bond is shown as *Tab. 3.4* below,

Tab. 3.4 Calculation volatility in value due to credit qulity changes

State at year-end	μ_j	p_j (%)	$p_j\mu_j$	$\Delta V_j = \mu_j - \mu_{Total}$	$p_i\mu_i^2 - \mu_{Total}^2$	σ_i	$(\mu_i^2 + \sigma_i^2) - \mu_{Total}^2$
AAA	109.35	0.02	0.02	2.28	0.0010	0	0.0010
AA	109.17	0.33	0.36	2.10	0.0146	0	0.0146
A	108.64	5.95	6.46	1.57	0.1472	0	0.1472
BBB	107.53	86.93	93.48	0.46	0.1847	0	0.1847
BB	102.01	5.30	5.41	-5.06	1.3589	0	1.3589
B	98.09	1.17	1.15	-8.98	0.9444	0	0.9444
CCC	83.63	0.12	0.10	-23.44	0.6596	0	0.6596
Default	53.80	0.18	0.10	-53.27	5.1078	26.86	6.4065
	Mean = 107.07				$\sigma_{Total}^2 = 8.42$ $\sigma_{Total} = 2.90$		$\sigma_{Adjusted-total}^2 = 9.72$ $\sigma_{Adjusted-total} = 3.12$

Source: Own calculation.

In summary, as calculation shown in *Tab. 3.4*, we can see that the standard deviation of our example is *€3*.12. It is a little bit higher than the non-adjustment standard deviation which is *€2.9*. Thus, the possibility of default plays a very important role in risk management of portfolio by incresing its credit risk.

Moreover, there is another useful way by calculating percentile level as a measure of credit risk. Now we determine the 1st percentile level for bond with probability 1% to measure credit risk. In this way, we concentrate on the cumulative probability starting from the bottom of the rating class, default and moving upwards to the AAA rating class. The meaning of 1st percentile level is that the value at which from bottom to up total first becomes equal to or greater than 1%. The cumulative probabilities and present value of bond a year from now can be shown as following *Tab. 3.5*.

Tab. 3.5 Value and cumulative probabilities

State at year-end	Difference of value from mean	Probabilities	Cumulative probabilities	Present value a year from now
Default	-53.27	0.18%	0.18%	53.80
CCC	-23.45	0.12%	0.30%	83.63
B	-8.99	1.17%	1.47%	98.09
BB	-5.07	5.30%	6.77%	102.01
BBB	0.46	86.93%	93.70%	107.53
A	1.57	5.95%	99.65%	108.64
AA	2.10	0.33%	99.98%	109.17
AAA	2.28	0.02%	100.00%	109.35

Source: Own calculation.

Illustrated in *Tab. 3.5*, when a BBB-rated obligor degrades to B-rated, the cumulative probability is firstly higer than 1%. In this case, the present value of portfolio a year from now is €98.09 and this is €8.99 below expected value. If we use 95% confidence level, we should focus on the rating class that the cumulative probability firstly exceed 5% which is €102.01, BB-rate and this is €5.07 below the expected value. Comparing these two confidence level, downgrading to B-rated is more risky than downgrating to BB-rated and it can be represented from diference of value from expected value that €8.99 is much higher than €5.07.

3.3.2 Portfolio risk calculation

In practice, it is more propular when there are more than one assets i one portfolio. In this case, we should focus more on the risk caluclation of portfolios that contains more than one bond. Here in this section, the discussion focus on the possibility of calculation of a two credit exposures' portfolio with an example portfolio consisting of the following two specific bonds,

- bond 1: BBB-rated, senior secured, 6% annual coupon, five-year maturity,
- bond 2: AA-rated, senior unsecured, 5% annual coupon, four-year maturity.

To estimate the portfolio credit exposure, the process is the same as steps we have mentioned above, but in greater detail. And we can see that the first example is as same as the example we have discussed above section thus it is only necessary to calculate the second bond with the same steps. And then, we have to estimate the risk brought by the effects of non-zero credit quality correlations in additional, which can be found as right-hand-side of frammework in *Fig. 3.5*. To calculate the correlation, we have to estimate joint likelyhoods in the credit quality co-movement. Generally speaking, there are three steps here in the process,

- step 1: calculation of joint probabilities,
- step 2: calculation of portfolio credit risk,
- step 3: marginal risk.

Step 1: calculation of joint probabilities

As we have discussed, there are 8 possibilities of migration for each the credit quality of an obligor in one year. Thus, there should be eight times eight which is sixty-four scenarios when there are two bonds in the portfolio. The simplest way to calculate joint possibility ($p_{i,j}$) of co-movement of credit quality of two obligors is the combination of two possibility of changes in credit quality of obligor. It can be expressed as follows:

$$p_{i,j} = p_i \cdot p_j, \tag{3.19}$$

Where p_i means the chance a BBB-rated convert to i-rated obligor and p_j means the chance an A-rated bond convert to j-rated obligor.

As an example of the situation that both two obligors remains their rating, the likehoods that the BBB-rated bond remains BBB is 86.93% and the A-rated bond remains A is 91.05%[10]. The joint possibility can be,

$$79.15\% = 86.93\% \cdot 91.05\%.$$

Note that this equation is correctly only if there is no correlation between these two bonds. The results of joint migration possibility with zero-correlation can be represented as *Tab. 3.6* below.

[10] The data is from Standard & Poor's CreditWeek (15 April 96)

Tab. 3.6 Joint migration possibility with zero correlation (%)

		Obligor #2 (A)							
		AAA	AA	A	BBB	BB	B	CCC	Default
Obligor #1 (BBB)		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.01	0.30	0.02	0.00	0.00	0.00	0.00
A	5.95	0.01	0.14	5.42	0.33	0.04	0.02	0.00	0.00
BBB	86.93	0.08	1.97	79.15	4.80	0.64	0.23	0.01	0.05
BB	5.30	0.00	0.12	4.83	0.29	0.04	0.01	0.00	0.00
B	1.17	0.00	0.03	1.07	0.06	0.01	0.00	0.00	0.00
CCC	0.23	0.00	0.01	0.21	0.01	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.16	0.01	0.00	0.00	0.00	0.00

Souce: Own calculation.

With the assumption of independence, we can see that the possiblility in a year-end which is illustrated in *Tab. 3.6* is centralized and the most possible situation of migration is that two obligors remains their initial rating whose joint possibility is 79.15%. However, the assumption of independence is not realistic due to same factors affecting, for instance economic cycle and interest rates are partly the reason of the rating changes and defaults of companies. There is another model can be used to link firm asset value to firm credit rating. It will be discussed in next section and the results of joint migration possibility with 30% correlation are shown as *Tab. 3.7* below.

Tab. 3.7 Joint possibilities with 0.30 asset correlation (%)

		Obligor #2 (A)							
		AAA	AA	A	BBB	BB	B	CCC	Default
Obligor #1 (BBB)		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.04	0.29	0.00	0.00	0.00	0.00	0.00
A	5.95	0.02	0.39	5.44	0.08	0.01	0.00	0.00	0.00
BBB	86.93	0.07	1.81	79.69	4.55	0.57	0.19	0.01	0.04
BB	5.30	0.00	0.02	4.47	0.64	0.11	0.04	0.00	0.01
B	1.17	0.00	0.00	0.92	0.18	0.04	0.02	0.00	0.00
CCC	0.23	0.00	0.01	0.09	0.02	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.13	0.04	0.01	0.00	0.00	0.00

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 38p.

Step 2: calculation of portfolio credit risk

Moreover, we can also calculate the present value of A-rated bond a year from now with the same forward rate we have used in *Chapter 3.3.1*. Then the total present value of portfolio a year from now can be determined with adding the present value of BBB-rated bond we have already estimated and the results can be represented as following *Tab. 3.8*.

Tab. 3.8 All possible 64 year-end values for a two-bonds portfolio (€)

Obligor #1 (BBB)		Obligor #2 (A)							
		AAA	AA	A	BBB	BB	B	CCC	Default
		105.84	105.70	105.30	104.43	100.43	97.36	83.94	51.53
AAA	109.35	215.19	215.06	214.65	213.78	209.78	206.72	193.29	160.88
AA	109.17	215.01	214.88	214.47	213.60	209.60	206.54	193.11	160.70
A	108.64	214.48	214.35	213.94	213.07	209.07	206.01	192.58	160.17
BBB	107.53	213.37	213.23	212.83	211.96	207.96	204.89	191.47	159.06
BB	102.01	207.85	207.71	207.31	206.43	202.43	199.37	185.95	153.54
B	98.09	203.93	203.79	203.39	202.51	198.51	195.45	182.03	149.62
CCC	83.63	189.47	189.33	188.93	188.05	184.05	180.99	167.57	135.16
Default	53.80	159.64	159.50	159.10	158.23	154.23	151.16	137.74	105.33

Source: Own calculation.

Next, we can calculate the expected value μ and standard deviation σ of all 64 possible values according to *Equation (3.11)* and *(3.12)*, and the calculation can be expressed as follow,

$$\mu = \sum_{i=1}^{64} p_i \mu_i = €212.45,$$

$$\sigma = \sqrt{\sum_{i=1}^{64} p_i \mu_i^2 - \mu^2} = €3.35.$$

Step 3: marginal risk

It is a significant problem to make a decision that holding a bond in or not within a portfolio. The calculation of marginal risk is going to estimate the marginal increase to the portfolio to the portfolio risk that would be created by adding a new bond.

According to the definition, we can calculate the marginal risk of our example of stand-alone credit risk and two-bond portfolio credit risk which is €3.12 and €3.35, respectively. The marginal standard deviation of adding a new A-rated bond is the difference between €3.12 and €3.35 which is €0.23. Furthermore, we can calculate the credit risk of A-rated bond which is €2.49, and we can see the marginal standard deviation is much lower than the credit risk of a stand-alone portfolio with a A-rated bond. The reason may be the diversification that is in turn caused by the fact that the year-end values of the individual bonds are not perfectly correlated.

As we have discussed above, the marginal risk can be estimated in the form of standard deviation and it can be extended to percentile levels to measure marginal risk. For stand-alone portfolio, the expected value is €107.07 and with a 1st percentile value of €98.09. This percentile level is €8.99 below the mean. After adding an A-rated bond, the portfolio has an expected value is €212.45 and a 1st percentile value of €202.43 with the likelihoods of all values less than this sum to 1%. This is €10.02 below expected value. Thus, the marginal risk in the form of percentile level is the difference between €10.02 and €8.99, which equals to €1.03. Moreover, we can calculate the 1st percentile value of stand-alone of A-rated bond, which is €4.74 below the mean value of €105.17. Comparing maginal risk with the stand-alone risk of A-rated bond, we can see that marginal risk (€1.03) is lower than stand-alone risk of A-rated bond (€4.74). It can be also regarded as the effect of diversification.

3.3.3 Credit quality correlation

To estimated joint possibility with correlation, there are three steps mentioned by CreditMetrics Technical Document, which is,

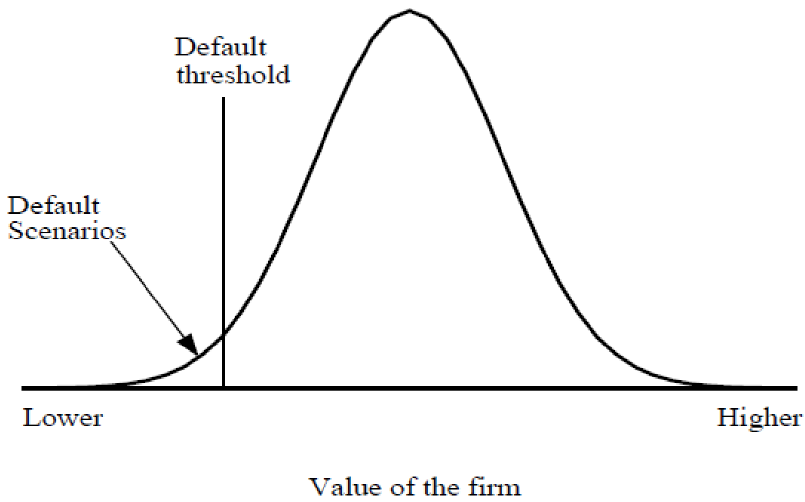
- step 1: asset value model, a modified version of the Merton model, is to estimate not only defaults but also migrations depend on changes in the value of corporate asset,
- step 2: estimates the correlation between the asset value model of two obligors,
- step 3: calculates joint possibility matrics based on the correlation we have estimated.

Step 1: asset value model

Asset value model is a modified version of the Merton model which we have already discussed in *Chapter 3.2.3* and it give possible to estimate probability of migration towards to different credit rating and defaults with company's assets.

Let us begin with Merton model. As we have mentioned, the main objective of Merton model is to analyze whether the company will survive or default in the future with the help of linking its company assets to value of option. Here, we extend the model to a multinominal case which means it includes defaults and migrations between different ratings. If we focus on the rightmost side of *Fig. 3.4*, there is a probability distrobution of all possible future firm's assets and if the future value is lower than the value of debt, there is a probability of default which we refer to as the default threshold. *Fig. 3.6* shows more details about the model of firm value and its default threshold,

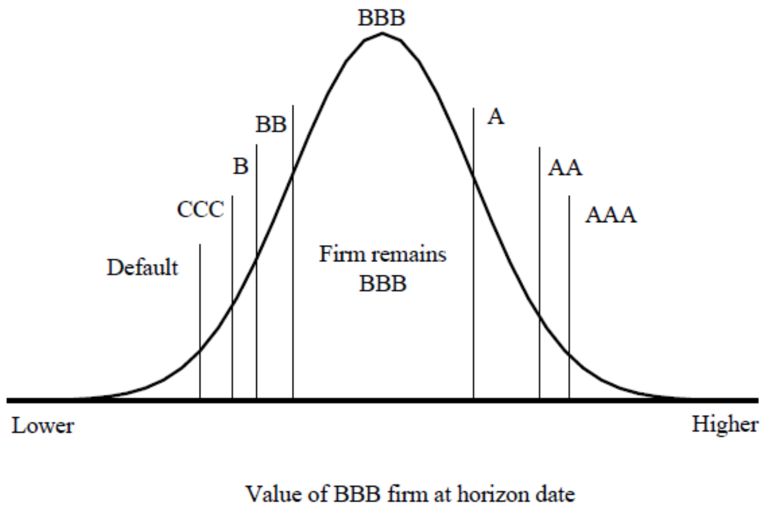
Fig. 3.6 Model of firm calue and its default threshold



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J.P.Morgan, 1997. 37p.

If the future value of firm is lower than default threshold, it is impossible from the firm to statisfy its debt obligation and it results in default. However, the estimation of probability of default here is not a big deal in CreditMetric model because one of the assumption of model is that there is always a credit rating assigned for each obligor. We need to extend it to include rating changes. By extension, we link default threshold to credit rating migration thresholds and the details can be presented as *Fig. 3.7* below.

Fig. 3.7 Model of firm value and generalized credit quality thresholds



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J.P.Morgan, 1997. 38p.

Illustrated in *Fig. 3.7*, the default-rating threshold is the same as the default threshold in *Fig. 3.6*, just assigning credit rating to each value of companies at time horizon. If the future value assets are below to this threshold, there is a result of default. However, if the future assets are between the CCC-threshold and BB-threshold, it may cause banks' analysts to downgrates the company to BB-rated. In this case, there is a easy mapping that credit rating class relatives to assets value if there is the specific number on horizontal axis and vertical axis. Thus, the only thing we need to do is to measure the changes in asset value in order to describe its credit rating movement. To that end, we need to assert another two parameters, which is the mean of asset value μ and standard deviaiton of asset returns σ due to the probability disstibution of asset return in the future is normal which is assumed by Merton model. Later, we may now establish a connection between the asset thresholds and the transition probabilities for company. Let us use Z_{def}, Z_{CCC}, Z_{BBB} and so on, to represent the asset return threshold, we can compute the probability, of a BB-rated for example, that each of these events occur and associated with the probability of migration, the results can be shown in following *Tab. 3.9*,

Tab. 3.9 One year transition probability for a BB rated obligor

Rating	Probability from the transition matrix (%)	Probability according to the asset value model
AAA	0.03	$1 - \Phi({Z_{AA}}/{\sigma})$
AA	0.14	$\Phi\left({Z_{AA}}/{\sigma}\right) - \Phi({Z_{A}}/{\sigma})$
A	0.67	$\Phi\left({Z_{A}}/{\sigma}\right) - \Phi({Z_{BBB}}/{\sigma})$
BBB	7.73	$\Phi\left({Z_{BBB}}/{\sigma}\right) - \Phi({Z_{BB}}/{\sigma})$
BB	80.53	$\Phi\left({Z_{BB}}/{\sigma}\right) - \Phi({Z_{B}}/{\sigma})$
B	8.84	$\Phi\left({Z_{B}}/{\sigma}\right) - \Phi({Z_{CCC}}/{\sigma})$
CCC	1.00	$\Phi\left({Z_{CCC}}/{\sigma}\right) - \Phi({Z_{def}}/{\sigma})$
Default	1.06	$\Phi({Z_{def}}/{\sigma})$

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J.P.Morgan, 1997. 87p.

Note that Φ denotes the cumulative distribution for the standard norminal distribution. For example, the probability accroding to asset value model equals to the probability of downgrades to default which is 1.06%, the caculation of Z_{def} can be,

$$Pr\{Default\} = Pr\{Value < Z_{def}\} = \Phi\left({Z_{def}}/{\sigma}\right) = 1.06\%$$

$$Z_{def} = \Phi^{-1}(1.06\%) \cdot \sigma = -2.30\sigma$$

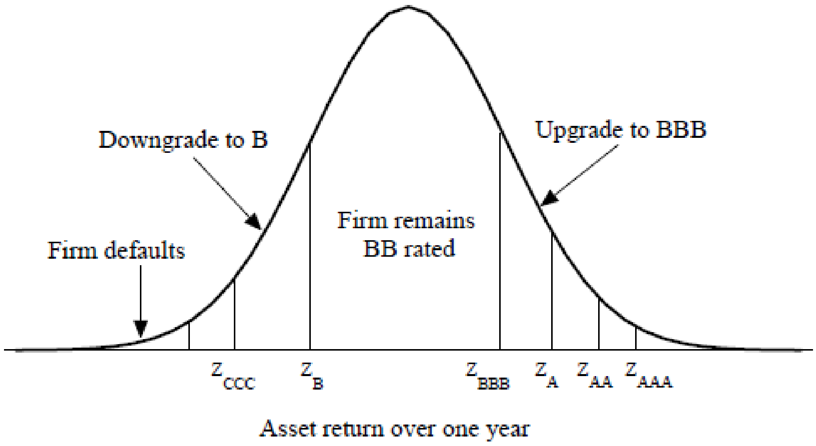
In the same way, we can obtain other values of asset threshold in *Tab. 3.9*.

Tab. 3.10 Threshold values for asset return for a BB-rated obligor

Threshold	Value
Z_{AA}	3.43σ
Z_{A}	2.93σ
Z_{BBB}	2.39σ
Z_{BB}	1.37σ
Z_{B}	-1.23σ
Z_{CCC}	-2.04σ
Z_{def}	-2.30σ

The results can be shown in graphical as following *Fig. 3.8* which representing a multinominal Merton model with default and migrations of this compony.

Fig. 3.8 A multinominal Merton model with default and migrations



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J.P.Morgan, 1997. 88p

As shown in *Fig. 3.8*, if the future asset value is between -1.23σ (Z_B) and 2.39σ (Z_{BBB}), the changes is not enough to justify a rating change, and firm remains in BB. If the asset value is higher than 2.39σ, the obligor will upgrade to BBB-rated. Note that, there is usually no asset threshold for AAA-rated due to the fact that the obligor will upgrade to AAA as long as asset value higher than Z_{AA} which is 3.43σ. In a same way, we can calculate the threshold value of AA-rated bond and the results can be sown as following *Tab. 3.11*.

Tab. 3.11 Transition possibilities and asset returen thresolds for AA rating

Rating	Probability from the transition matrix (%)	Probability according to the asset value model	Cumulative probability (%)	Thresholds
AAA	0.09	$1-\Phi({}^{Z_{AA}}/_{\sigma})$	100.00	
AA	2.27	$\Phi\left({}^{Z_{AA}}/_{\sigma}\right)-\Phi({}^{Z_{A}}/_{\sigma})$	99.30	$3.12\sigma'$
A	91.05	$\Phi\left({}^{Z_{A}}/_{\sigma}\right)-\Phi({}^{Z_{BBB}}/_{\sigma})$	8.65	$1.98\sigma'$
BBB	5.52	$\Phi\left({}^{Z_{BBB}}/_{\sigma}\right)-\Phi({}^{Z_{BB}}/_{\sigma})$	0.86	$-1.51\sigma'$
BB	0.74	$\Phi\left({}^{Z_{BB}}/_{\sigma}\right)-\Phi({}^{Z_{B}}/_{\sigma})$	0.23	$-2.30\sigma'$
B	0.26	$\Phi\left({}^{Z_{B}}/_{\sigma}\right)-\Phi({}^{Z_{CCC}}/_{\sigma})$	0.17	$-2.72\sigma'$
CCC	0.01	$\Phi\left({}^{Z_{CCC}}/_{\sigma}\right)-\Phi({}^{Z_{def}}/_{\sigma})$	0.03	$-3.19\sigma'$
Default	0.06	$\Phi({}^{Z_{def}}/_{\sigma})$	0.01	$-3.24\sigma'$

Source: Own calculation.

At this point, we have discussed the motion of each obligor individually accoding to it asset value process. We would combine the movement of credit rating of two obligors under the assumption that they are correlated and normally distributed. Then, we have the covariance matrixs for the bivariate normal distribution with correlation ρ, which is:

$$\Sigma = \begin{pmatrix} {\sigma_{BBB}}^2 \rho\sigma_{BBB}\sigma_{AA} \\ \rho\sigma_{BBB}\sigma_{AA}{\sigma_{AA}}^2 \end{pmatrix}. \tag{3.20}$$

Step #2: Estimation of correlation

CreditMetric estimates the correlation between asset returns with the help of the correlations between the returns of stocks index of various counties and industries. And it also assume that the asset returns of each company are determined by various of systematic risk factors and idiosyncratic factor. The idiosyncratic factor has no correlation with other risk factors and it is a specific factor to the individual company. Therefore, we can calculate the correlation between systematic factors later with the assumpition that the asset value return r_j of a company j is a linear combination of all systematic fators I_k and a specific term ε. In detail:

$$r_j = \beta_{1,j}I_1 + \beta_{2,j}I_2 + \cdots + \beta_{n,j}I_n + \delta_j\varepsilon_j, \tag{3.21}$$

Where $\beta_{k,j}$ means that the weight of factor I_k in explaining the asset return of company j, while δ_j denotes the weight of the idiosyncratic component.

The systematic risk factor I_k can be various variables in every industries and countries. For instance, it can link to the performance of the automotive or algriculture industries, or to that of the Chinese or American economy. Let us assume the asset value return of two obligors can be expressed as followe,

$$r_A = \beta_{1,A}I_1 + \beta_{2,A}I_2 + \delta_A\varepsilon_A,$$

$$r_B = \beta_{3,B}I_3 + \beta_{4,B}I_4 + \delta_B\varepsilon_B,$$

Concequencely, the correlation between the asset returns of company A and company B can be as follows,

$$\rho_{A,B} = \beta_{1,A}\beta_{3,B}\rho_{1,3} + \beta_{1,A}\beta_{4,B}\rho_{1,4} + \beta_{2,A}\beta_{3,B}\rho_{2,3} + \beta_{2,A}\beta_{4,B}\rho_{2,4}. \tag{3.22}$$

Where ρ_s denotes correlation between systematic factors and the correlations with idiosyncratic factors are zero.

Step 3: Estimation of jiont migration possibilities

Next we compute the probability of co-movement of two obligors with correlation. As an exmple of two obligors remaining its initial rating, in this case, the changes of asset return for BB-rated obligor is between Z_B and Z_{BB} and the changes of asset return for A-rated obligor is between Z'_{BBB} and Z'_A. The calculation can be expressed as follow,

$$Pr\{Z_B < r_A < Z_{BB}, Z'_{BBB} < r_B < Z'_A\} = \int_{Z_B}^{Z_{BB}} \int_{Z'_{BBB}}^{Z'_A} f(r_A, r_B; \Sigma)(dr_B)(dr_A). \tag{3.23}$$

Where $f(r, r'; \Sigma)$ is the density function[11] for the bivariante normal distribution with covariance matrix Σ, r and r' represent the values that two asset returns may take on within the specified intervals.

3.4 Regulation of capital requirements

In this chapter, there will be some regulations of caital requrement of banks under Basel Accord. We would focus on the calculation of capital requrements under Basel I Accord, enhenced version of calculation of capital reqirements by several approaches under Basel II Accord and the measures to strengthen the regulation, supervision and risk managements of banks after financial crisis under Basel III Accord.

Basel Accord was firstly presented in 1988 by Basel Committee whose initial name is the Committee on Banking Regulations and Supervisory Practices, established by the central bank Governors of the Group of Ten[12] countries at the end of 1974[13], to give a standard for the regulation of banking system in order to increase the financial stability by improving the quality of banking surpervision worldwide. These series of international standards for bank regulation

[11] Density function can be expressed as: $f(r, r'; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{x^2 - 2\rho r r' + y^2}{2(1-\rho^2)}}$.

[12] G10 Governors includes Belgium, Canada, France, Italy, Germany, Japan, Nertherlands, Sweden, the United Kingdom, and, the United states.

[13] Source: history of the Basel Committee, available on: https://www.bis.org/bcbs/history.htm

are most notably its landmark publications of the accords under capital adequacy which are commoly known as Basel I Accord, Basel II Accord and, most recently, Basel III Accord. These three accords are introduced in the next sections, respectively.

3.4.1 Basel I Accord

In the early 1980s, the capital ratios of the main international banks were deteriorating with a growing of international risk and a onset of Latin American debt crisis heightened the Committee's concern. With the support of G10 Governor, committee published a standard of a measurement of capital adequacy in 1988 includeng a weighted approach to the measurement of risk. The standard named "International Convergence of Capital Measurement and Capital Standards", commonly referred to as the Basel Capital Accord.

Under Basel Capital Accord, firstly, they described the constituents of capital. There are two types of regulatory capital,

- tier 1: Core capital (basic equity) includeds permanent shareholders' equity and disclosed reserves with the deductions of goodwill. Permanent sharehodels' equity contains common stock, perpetual non-cumulative prederence shares and disclosed reserve includes created of increased by appropriations of retained earnings or other surplus such as share premiums, retained profit, general reserves and legal reserves.
- tier 2: Supplementary capital includes undisclosed reserve, revaluation reserves, general provisions, hybrid debt capital instruments and subordinated term debt. Undisclosed reserve consist of the part of the accumulated after-tax surplus of retained profits which banks in some countries may be permitted to maintain as an undisclosed reserve. Revaluation reserves includes the revalution of banks' fixed assets such as premise under laws in some countries and the hidden value resulting from long-term holdings of equity securities valued in the balance sheet at the historic cost of acquisition. Hybrid capital instruments means the vatious instruments which includes the combination of equity capital and debt. *Subordinated term debt includes conventional unsecured subordinated debt capital insruments with a minimum*

original fixed term to marturity of over five years and limited life redeemable preference shares[14].

There are some limitations of tier 1 and tier 2 capitals. Firstly, tier 1 capital is higher or equals to tier 2 capital. Secondly, subordinated debt should be lower than a half of tier 1 capitals. Thirdly, revaluation reserves which from the laten gains on unrealised securities will be subjuct to a disount of 55%.

Secondly, Committee considers that there should be different weighted risk ratio in which capital is related to different categories of asset. These weighted risk ratio can be used to calculate risk-werighted-asset (RWA) to further calculate capital adequacy ratio by risk-weighted approach of capital measurement. There are four categories of weghted risk ratio including 0%, 20%, 50% and 100%. The lower the weights, the lower the risk. For instance, cash is regarded as non-risk assets assigned with 0% of wighted risk ratio. The following *Tab. 3.12* presents the specific items in each weights.

[14] BASEL COMMITTEE ON BANKING SUPERVISION: *Internaltional convergence of capital instrumrnt and capital standards,* July 1988. 20p.

Tab. 3.12 Risk weights by category of on-balance-sheet asset

0%	a) Cash b) Claims on central governments and central banks denominated in national currency and funded in that currency c) Other claims on OECD central governments and central banks d) Claims collateralized by cash of OECD central-government securities or guaranteed by OECD central government
20%	a) Claims on multilateral development banks (IBRD, IADB, AsDB, AfDB, EIB) and claims guaranteed by, or collateralized by securities issued by such banks b) Claims on banks incorporated in the OECD and loans guaranteed by OECD incorporated banks c) Claims on banks incorporated in countries outside the OECD with a residual maturity of up to one year and loans with a residual maturity of up to one year guaranteed by banks incorporated in countries outside the OECD d) Claims on non-domestic OECD public-sector entities, excluding central government, and loans guaranteed by such entities e) Cash items in process of collection
50%	a) Loans fully secured by mortgage on residential property that is or will be occupied by the borrower or that is rented
100%	a) Claims on the private sector b) Claims on banks incorporated outside the OECD with a residual maturity of over one year c) Claims on central governments outside the OECD (unless denominated in national currency - and funded in that currency) d) Claims on commercial companies owned by the public sector e) Premises, plant and equipment and other fixed assets f) Real estate and other investments (including non-consolidated investment participations in other companies) g) Capital instruments issued by other banks (unless deducted from capital) h) all other assets

Source: BASEL COMMITTEE ON BANKING SUPERVISION: Internaltional convergence of capital instrumrnt and capital standards, July 1988. 21p.

Then the risk-weighted assets form N items can be computed as follow:

$$RWA = \sum_{i=1}^{N} w_i \cdot EAD_i \,, \tag{3.24}$$

Where w_i is the risk weight of ith item and EAD_i is the exposure at default of ith item.

At last, Committee presented a series of target standard ratio of capital adequacy, which can be expressed as follows,

$$Tier\ 1\ ratio = \frac{Tier\ 1\ capital\ element}{RWA} \geq 4\%, \tag{3.25}$$

$$Capital\ adequacy\ ratio = \frac{Tier\ 1\ capital + Tier\ 2\ capital}{RWA} \geq 8\%, \tag{3.26}$$

The capital adequacy ratio is expressed as a common minimum standard which international banks in member countries would be expected to observe form the end of 1992. However, this equation presented here only reflects credit risk and there was a modified version of capital adequacy containing market risk by 1996 Market Risk Amendment. The modified version of capital adequacy ratio can be given by,

$$Capital\ adequacy\ ratio = \frac{Tier\ 1\ capital + Tier\ 2\ capital}{RWA + (12.5 \cdot CR_m)} \geq 8\%. \tag{3.27}$$

Where CR_m is capital requrement for market risk.

3.4.2 Basel II Accord

In June 2004, the Committee released a revised capital framework, generally known as Basel II Accord. Under Basel II, the Committee refined the framework to address a new risk, which is operational risk, rather than credit risk and market risk. Furthermore, there are also several approaches for the measurement of credit risk, market risk and operational risk, specifically. It avoid the significant weakness of Basel I that assigning same risk-weight to different rating class loans. The revised framework of Basel Accord consists of three pillars:

- pillar 1: minimum capital for credit, market and operational risk,
- pillar 2: supervisory review for risk capital,
- pillar 3: market discipline.

Pillar 1: minimum capital for credit, market and operational risk

The first pillar of Basel II is to give a standardized rule of credit risk, market risk and operational risk with the regulatory capital and risk-weighted assets. It is a first attempt to assign a regulatory capital charge to the management of operational risk. Under Basel I, the regulatory capital divided into tier 1 and tier 2, assigning regulatory capital charge to the management of credit risk. Under the press release in 1996 Market risk Amendment, the Committee associated market risk with regulatory capital and, in the meanwhile, the Amendment presents another capital which is tier 3. Tier 3 capital can be used with the discretion of their national authority, consisting of short-term subordinated debt and tier 3 capital are limited to 250% of a banks' tier 1 capital that is required to support markte risk. The calculation of total RWA is also changed with addtional capital requirement for operational risk and it can be expressed as:

$$RWA_{Total} = RWA_c + 12.5 \cdot (CR_m + CR_o), \tag{3.28}$$

Where RWA_c means the risk-weighted -assets for credit risk, CR_m denotes the capital requirement for market risk and CR_o is the capital requirement for operational risk.

Thus, the capital ratio can be computed as follow,

$$Capital\ adequacy\ ratio = \frac{Tier\ 1 + Tier\ 2 + Tier\ 3}{RWA_c + 12.5 \cdot (CR_m + CR_o)} \geq 8\%, \tag{3.29}$$

Note that the tier 3 capital can be only used to measure capital requirement for market risk. In this case, for credit risk, the eligible tier 1 and tier 2 capital must be higher than $8\% \cdot RWA_c$, however, for market risk, the eligible tier 1, tier 2 and tier 3 should be higher than CR_m.

Moreover, Basel II defines several approaches to estimate the bank's riskiness for each tisk type. It can be summarized as following *Tab. 3.13*.

Tab. 3.13 Methods for calculation capital according to Basel II

	Credit risk	Market risk	Operational risk
Approaches	• Standardized approach • Foundation internal ratings-based rating (IRB) approach • Advanced IRB approach	• Standardized Approach • Internal model approach	• Basic indicator approach • Standardized approach • Advanced measurement approach
Result	Risk-weighted asset value for credit risk	Market risk capital charge	Operational risk capital charge

Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 203p.

Standardized approach calculation of capital requirement of credit risk

The standardized approach is according to the calculation of risk-weight-asset mentioned in *Equation (3.18)*. However, the weighted-risk-ratio are divided into several parts accordint to both its rating class and its obligors. There is an example of risk weights shown as *Tab. 3.14* below.

Tab. 3.14 Capital requirement risk weights under Basel II

	Government	Public sector	Banks	Corporations
AAA to AA	0%	20%	20%	20%
A+ to A-	20%	50%	50%	50%
BBB+ to BBB-	50%	100%	100%	100%
BB+ to B-	100%	100%	100%	100%
B+ to B-	100%	150%	150%	150%
Below B-	100%	150%	150%	150%
Unrated	100%	100%	100%	100%

Source: BIS.

Let us assume that our bank now have some net receivables including *€1*00, A+-rated government, *€1*50, B+-rated Banks, *€2*00, A-rated corporeates and *€2*50, BB-rated public sector. And the risk weights of these receivables are 20%, 150%, 50% and 100%, respectively.

Thus, the risk-weighted-assets and capital requirement for credit risk can be computed as follow,

$$RWA = 100 \cdot 20\% + 150 \cdot 150\% + 200 \cdot 50\% + 250 \cdot 100\% = 595,$$

$$CR_c = \frac{RWA}{12.5} = 47.6.$$

Internal rating-based (IRB) approach

The difference between standardized approach and foundation internal raing-based approach is the calculation of risk-weighted-asset. Here in this approach, the calculation of risk-weighted appraoch is not followed by *Equation (3.18)* but some other parameters such as probability default, loss given default and exposure at default. More specifically, banks are allowed to use internal credit rating model to calculate the probability of default and the loss given default and exposure at default should be permitted by local regulator. And for advanced IRB approach, all the parameters can be estimated under internal quantative model of banks.

Pillar 2: supervisory review for risk capital

The aim of the second pillar is to complement and strengthen the first pillar by establishing a prudential supervision process. It covers all the risk in pillar 1 and adds some other considerations:

- banks should hold some economic capital in order to survive during crisis to cover risks classified in Pillar 1. The economic capital has been already discussed in *Chapter 3.1* and it give supports to cover unexpected loss during time of distress,
- banks need to establish a governance structure to enhance the internal surpervision and oversight from the board of directors and senior management,
- there should be a rating process of banks by banking supervisor to measure the risk level of banks themselves.

Pillar 3: market discipline

The aim of the third pillar is to strengthen disclosure and encourage good banking practices by means of the effective usage of market discipline. It require that banks need to disclose their financial conditions to depositors, shareholders and other interested parties. The requirement of high transparency of financial condition helps those interested parties to make decisions as a result of seeking the interest. Transparency, or disclosure, measures the degree of banks to reveal its assets, liabilities and other internal workings.

3.4.3 Basel III Accord

Under Basel II Accord, the measurement of risk relies greatly to credit rating from rating agencies leading to banks cannot figure out risky assets. For example, asset-based securities whose underlying assets includes the mortgage, loans and other sucurities appeared and are traded in secondary market. This type of securities are rated by rating agencies and have the same rating class mode such as AAA, AA and so on which is the same as the rating class of its underlying assets. It results in that these asset-based securities are considered same risk level with its underlying assets by investors while assest-based securities are obviously more risky than its underlying assets. Thus, the financial crisis revealed the whole world in 2008 because of deregulation and globalization of financial market. Basel III Accord was developed in response to the deficiencies in financial regulation after financial crisis 2008. It extended capital requirement by increasing banks' liquidity and decreasing banks' leverage. It was agreed on Basel Commettee in Nov. 2010 and was scheduled to be introduced from 2013 untile 2015. However, implementation was extended repeatedly to the end of Mar. 2019.

Under Basel III, the rules required banks to fund themselves with 4.5% of common equity capital of risk-weighted assets and it should be maintained in 2015 by the bank. This ratio is calculated as follows,

$$Common\ equity\ tier\ 1\ (CET1)\ ratio = \frac{CET1}{RWAs} \geq 4.5\%, \tag{3.30}$$

Also, the tier 1 ratio increased from 4% in Basel II to 6% and it should be applicable in 2015. Tier 1 ratio consists of common equity tier 1 ratio and additional tier 1 which accounts to 1.5%. Futhermore, Basel III introduce two additional capital buffers. One is a mandatory capital consevation buffer. It equals to 2.5% of RWAs. In total, banks are required to hold a total of 7% buffer capital ratio considering a 4.5% CET1 capital ratio. Another capital buffer is a discretionary counter-cyclical buffer. It allows national regulators to require up to an additional 2.5% of capital during periods of high credit growth. The level of this buffer ranges between 0% and 2.5% of RWA and must be met by CET1 capital.The summary of Basel III phase-in arrangements presented as *Tab. 3.15*.

Tab. 3.15 Basel III phase-in arrangements (%)

	Phases	2013	2014	2015	2016	2017	2018	2019
Capital	Leverage ratio			a[15]			b[16]	
	Minimum common equity capital ratio	3.5	4.0	4.5				**4.5**
	Capital conservation buffer				0.625	1.25	1.875	**2.50**
	Minimum common equity plus capital conservation buffer	3.5	4.0	4.5	5.125	5.75	6.375	**7.0**
	Phase-in of deductions from CET1		20	40	60	80	100	**100**
	Minimum Tier 1 capital	4.5	5.5	6.0				**6.0**
	Minimum total capital	8.0						**8.0**
	Minimum total capital plus conservation buffer	8.0			8.625	9.25	9.875	**10.5**
	Capital instruments that no longer qualify as non-core Tier 1 capital or Tier 2 capital	Phased out over 10-year horizon beginning 2013						
Liquidity	Liquidity coverage ratio - minimum requirement			60	70	80	90	**100**
	Net stable funding ratio						c[17]	

Source: BIS.

Accord introduced a munimun leverage ratio as well. It is a non-risk-based leverage ratio and is calculated by dividing tier 1 capital by the bank's avrage total consilidated assets consisting of exposures of all assets and non-balance sheet items. It can be expressed as follow,

$$minimun\ leverage\ ratio = \frac{Tier\ 1\ capital}{Total\ exposures} \geq 3\%, \tag{3.31}$$

[15] Parallel run 1 Jan 2013 - 1 Jan 2017, Disclosure starts 1 Jan 2015.

[16] Migration to Pillar 1.

[17] Introduce minimum standard.

For liquidity requirement, the liquidity coverage ratio (LCR) was supposed to require a bank to hold sufficient high-quality liquid assets to cover its total net cash outflows over 30 days. It can be expressed as,

$$LCR = \frac{Stock\ of\ high - quality\ liquid\ asset}{Total\ net\ cash\ outflows\ over\ the\ next\ 30\ calender\ days} \geq 100\%, \qquad (3.32)$$

Assets which can be easily and immediately converted into cash at little or no loss of value are considered as high-qulity assets. There are some characteristics of high-qulity assets, in general, fundamental charactristic and market-related characteristics. Fundemental charactristic relates to basic characteristic of assets itself. It includes low credit and market risk, ease and certainty of valuation, low correlation with risky assets and listed on a developed and recognised exchange market. On the other hand, market-related characteristic is associated with the fluctuation of market, including active and sizable market, presence of committed market makers, low market concentration and flight to quality.

And the net stable funding ratio (NSFR) require that the available amount of stable funding (ASF) should be higher than the require amount of stable funding (RSF) over a one-year period of extended stress. The stable funding is defined as the total equities and liabilities that can be used to fund over one year horizon under condition of financial stress. The avalable stable funding (ASF) includes a bank's total capital, preferred stock with maturity over one year, liabilities with maturity of one year or greater and so on. Each types of available funding accounts a proportion of total ASF and total ASF is the sum of weighted amounts. *Tab. 3.16* below shows each ASF catagories and its factors.

Tab. 3.16 Components of availabe stable funding and associated ASF factors

ASF Factor	Components of ASF Category
100%	The total amount of capital, including both Tier 1 and Tier 2 as defined in existing global capital standards issued by the Committee
	The total amount of any preferred stock not included in Tier 2 that has an effective remaining maturity of one year or greater taking into account any explicit or embedded options that would reduce the expected maturity to less than one year.
	The total amount of secured and unsecured borrowings and liabilities (including term deposits) with effective remaining maturities of one year or greater excluding any instruments with explicit or embedded options that would reduce the expected maturity to less than one year. Such options include those exercisable at the investor's discretion within the one-year horizon.
90%	Stable non-maturity (demand) deposits and/or term deposits with residual maturities of less than one year provided.
80%	Less stable non-maturity (demand) deposits and/or term deposits with residual maturities of less than one year provided by retail and small business customers.
50%	Unsecured wholesale funding, non-maturity deposits and/or term deposits with a residual maturity of less than one year, provided by non-financial corporates, sovereigns, central banks, multilateral development banks and PSEs.
0%	All other liabilities and equity categories not included in the above categories.

As we all known, deposits are a large part of liabilities of banks. Thus, retail deposits can be also a type of stable funding. However, we can see in *Tab. 3.16* above, stable deposits and less stable deposits accounts different weights for calculation of ASF. The main difference between stable deposits and less stable deposits is run-off ratio. The run-off ratio of stable deposits is required at least 5% and, for less stable deposits, it is required at least 10%.

4 Determination of credit risk by selected model

After the descriptions of basic theory of credit risk models, we would like to assess the credit risk in the practice. In this chapter, the main goal is to estimate credit risk by different methods. In that case, the economic capital due to credit risk is estimated according to the real data from Frankfurt Stock Exchange under the CreditMetrics™ model and the capital requirement is calculated as well based on Basel Accord.

Here in Chapter 4, we divide it into four parts. Firstly, we describe the selected porfolio including ten different industries and credit rating obligors. Secondly, there are calculations of economic capital based on CreditMetrics™ model. The following behind is the calculation of capital requirement under Basel Accord by using standard approach (SA) and the foundation internal ratings-based approach (Foundation IBR). At last, the results are presented and explained in details.

4.1 Input data

The portfolio we selected here consists of ten different bonds trading in Frankfurt Stock Exchang whose obligors are publicly traded as well. For diversification, obligors we selected have different credit rating and are in different industries. In this case, assume that the total nominal value of portfolio is 10 million euro, the nominal value of each bonds is 1 million euro in average to avoid bias due to different nominal value assessing different bonds. All the information can be found in the official website of Frankfurt Stock Exchange and the date we collected these data is 1st March, 2019.

The input data mainly includes the nominal value of each bonds, coupon rate, market price, maturity and so on. Besides, all those bonds have a seniority Senior Unsecured in that they are not required to cover motgage loans. *Tab. 4.1* summarizes all the input data of portfolio.

Tab. 4.1 Basic information of individual bonds

	Rating	Coupon	Nominal value	Maturity	Market price	pcs.
BMW	A+	2.63%	2,000 €	1/2024	110.31%	500
Adidas	AA-	2.25%	1,000 €	10/2026	102.38%	1,000
HeidelbergCement	BBB-	1.50%	1,000 €	2/2025	102.22%	1,000
Deutsche Bank	BBB+	3.63%	1,000 €	9/2023	102.63%	1,000
Deutsche Telekom	BBB+	0.63%	1,000 €	12/2024	100.68%	1,000
Suedzucker	BBB	1.25%	1,000 €	11/2023	101.53%	1,000
Allianz	AA	3.50%	10,000 €	2/2022	110.32%	100
Deutsche Post	A-	1.00%	1,000 €	12/2027	101.01%	1,000
Daimler	A	1.38%	10,000 €	5/2028	100.96%	100
ThyssenKrupp	A-	3.13%	1,000 €	10/2025	101.12%	1,000

Source: Frankfurt Stock Exchnage.

As shown in *Tab. 4.1*, there are ten obligors with different ratings which are assessed by Standard & Poor's (S & P). The highest rating obligor is Allianz (AA), an insurance company from Germany and the lowest obligor is HeidelbergCement (BBB-), a multinational building material company from Germany. These bonds are denominated in Euro (*€)* and the nominal value varies from 10,000 to 1,000 euro resulting in different pieces of each bonds in the portfolio.

Because of different credit rating of each bonds, it is significant to figure out different default rate of each credit rating in Senior Unsecured class. The specific default rate by different rating is presented as *Tab. 4.2* below.

Tab. 4.2 The probability of default for different rating

Rating	PD	Rating	PD
AAA	0.0007%	**BBB-**	0.2747%
AA+	0.0022%	**BB+**	0.7117%
AA	0.0024%	**BB**	1.2581%
AA-	0.0044%	**BB-**	4.1917%
A+	0.0142%	**B+**	8.8480%
A	0.1075%	**B**	24.4180%
A-	0.2020%	**B-**	48.6187%
BBB+	0.2045%	**CCC**	-
BBB	0.2730%		

Source: Standard & Poor's

The recvery rate is 51.13% according to *Tab. 2.5* from Carty & Lieberman in Chapter 2 due to that each bond is Senior Unsecured class and the standard deviation is 25.45%. Thus, we can calculate the loss given default is 48.87% based on *Equation (2.1)*. Moreover, the transition matrix can be found in Annex 1 and this transition matrix is from Standard & Poor's.

4.2 Calculation of credit risk by CreditMetrics™

In this subchapter, we are going to estimate the economic capital by using CreditMetrics model. Accoding to the description of model, there are mainly four steps in the process. Firstly, we need to estimate the correlation of each obligors by generating a series of share price in one year time horizon. Then, there is the estimation of present value of each bond in each rating class by means of discounting forward yields. The forward yields curve in each rating acalss can be claculated by possibility migration matrix. Then, the Monte Carlo stimulation generate 30,000 random yields for each bonds and it is a way to obtain the value of the porfolio. At last, the results, expressed as economic capital, are calculated in certain confidence level.

4.2.1 Estimation of the correlation among bonds issuers

To estimate the correlation among bonds issuers, we focus on the market price of shares of each obligor. For avoiding bias, we choose one year time horizon of the value of shares on each trading day and it is from March 13rd, 2018 to March 12nd, 2019. *Tab. 4.3* presents the correlation matrix of ten obligors and the calculation is based on stock prices of each company in FSE which can be found in Annex 2.

Tab. 4.3 Correlations among individual issuers

	BMW	*ADIDAS*	*HEI.C*	*DB*	*DTE*	*SZU*	*Allianz*	*DP*	*Daimler*	*Thyssen*
BMW	1.00	0.32	0.54	0.07	-0.06	-0.08	-0.12	0.00	-0.04	0.03
ADIDAS	0.32	1.00	0.28	0.02	-0.12	-0.05	-0.08	-0.01	-0.05	-0.03
HEI.C	0.54	0.28	1.00	0.11	-0.02	-0.07	-0.01	0.03	0.00	0.03
DB	0.07	0.02	0.11	1.00	0.08	0.03	0.15	0.14	0.18	0.10
DTE	-0.06	-0.12	-0.02	0.08	1.00	0.14	0.48	0.30	0.35	0.22
SZU	-0.08	-0.05	-0.07	0.03	0.14	1.00	0.23	0.14	0.20	0.21
Allianz	-0.12	-0.08	-0.01	0.15	0.48	0.23	1.00	0.51	0.61	0.41
DP	0.00	-0.01	0.03	0.14	0.30	0.14	0.51	1.00	0.51	0.36
Daimler	-0.04	-0.05	0.00	0.18	0.35	0.20	0.61	0.51	1.00	0.52
Thyssen	0.03	-0.03	0.03	0.10	0.22	0.21	0.41	0.36	0.52	1.00

Source: Own calculation.

By means of analytical tools of MS Excel – Data/Data Analysis, we can obtain not only the correlation matrix, presented as *Tab. 4.3* above, but also the covariance matrix which can be found in Annex 3 from the market price of shares of each obligors. Both two matrix shows some relationships between each pair of bonds. As we discussed before, the obligors we selected are supposed to be in different industries, thus, t the correlations of each pair of bonds are quiet low to show this information. As example of Daimler and HerdelbergCement, whose correlation is zero, one is automotive corporate and another one focus on building materials which is less related industry with another. Moreover, the portfolio is not perfect divisified beacuse several pair of issuers shows much higher correlation which is higher than 0.5 from matrix due to two issuers operate in two highly related industries. For example, the correlation of HerdelbergCement and Thyssenkkrupp is 0.52 and Thyssenkkrupp also conduct bussiness with steel materials.

4.2.2 Calculation of the value of bonds

The second step is to calculate present values of each selected bonds. As we discussed in previous chapter, firstly, we need to calculate discount rate for each cash flow in the future. In consideration of different maturity of each bond, we need to conduct multiannual transition matrix in each year and the longest maturity date is in 2028 which means, in total, we need to conduct ten years' transition matrix in the future and cut out the default possibility parts. The initial annual transition matrix can be found in Annex 1 and the final results of multiannual transition matrixes are presented in Annex 4. The risk-free assets we use is EURO interest rate swap from 2019 to 2028 and the data can be found in official websit of Erste Group. Then we have forward discount rate in ten years according to *Equation (3.11)* and it is presented as following *Tab. 4.4*.

Tab. 4.4 Spot rate (IRS) and forward rate (f_n) from 2019 to 2025 (%)

Year	2019 1	2020 2	2021 3	2022 4	2023 5	2024 6	2025 7	2026 8	2027 9	2028 10
IRS	-0.24	-0.19	-0.11	-0.01	0.08	0.18	0.28	0.39	0.48	0.58
f_n	-0.24	-0.13	0.04	0.30	0.45	0.67	0.88	1.11	1.28	1.40

Source: Erste Group.

Now, we have possibility of default in ten years and forward discount rate. With the recocery rate, we can conduct the disount rate for each rating class in ten years based on the *Equation (3.15)*. The results of the forward discount rate can be found in Annex 5.

Later, the present values of each bond can be estimated according to *Equation (3.10)* and the below *Tab. 4.5* represents the present value of each bond in every rating category.

Tab. 4.5 Present values of bonds according to the rating categories (€)

Bond	BMW	ADIDAS	HEI.C	DB	DTE	SZU	Allianz	DP	Daimler	Thyssen
AAA	2,232	1,091	1,043	1,158	997	1,040	11,277	978	10,014	1,155
AA+	2,232	1,090	1,043	1,158	997	1,040	11,276	978	10,010	1,155
AA	2,232	1,090	1,043	1,158	997	1,040	11,276	978	10,009	1,155
AA-	2,231	1,090	1,043	1,157	997	1,039	11,275	978	10,005	1,154
A+	2,230	1,089	1,042	1,157	997	1,039	11,272	977	9,998	1,154
A	2,226	1,087	1,040	1,155	995	1,037	11,254	975	9,971	1,151
A-	2,227	1,087	1,040	1,155	995	1,038	11,259	974	9,969	1,151
BBB+	2,221	1,084	1,037	1,153	992	1,035	11,241	971	9,934	1,148
BBB	2,217	1,081	1,035	1,151	990	1,034	11,225	968	9,905	1,146
BBB-	2,206	1,075	1,029	1,146	985	1,029	11,179	963	9,844	1,140
BB+	2,207	1,075	1,029	1,147	986	1,030	11,195	962	9,839	1,140
BB	2,184	1,063	1,018	1,136	975	1,020	11,095	951	9,722	1,128
BB-	2,127	1,031	989	1,110	949	995	10,873	920	9,401	1,096
B+	2,086	1,010	969	1,089	930	976	10,685	902	9,217	1,075
B	1,994	966	926	1,042	888	933	10,235	862	8,817	1,028
B-	1,710	826	792	897	760	801	8,868	736	7,530	881
CCC	1,374	667	639	718	612	642	7,049	596	6,098	709

Source: Own calculation.

Tab. 4.5 summarizes all the present values of each bonds migrating from default to each rating category. We can see that the higher the final rating, the higher the present value. The cells highlighting with yellow color represents the present values of each bonds migrating from default to rating assigned at now.

4.2.3 Stimulation of the value of portfolio

Next, we would conduct Monte Carlo stimulation. Firstly, we need to generate a large number of random yields. Here, it can be achieved by using the function of MS Excel – Data/Data Analysis/Random Number Generation. The random yields should follow normal distribution N (0,1) and we generate in total 30,000 scenarios of each bonds. The whole scenarios are presented in Annex 6.

On the other hand, we can see that each pairs of issuers are not perfectly independent and the dependencies should be taken into consideration when stimulating. In this case, the upper triangular Cholesky decomposition matrix can be used as shown in *Tab. 4.6*.

Tab. 4.6 Cholesky decomposition matrix

	BMW	*Adidas*	*HEI.C*	*DB*	*DTE*	*SZU*	*Allianz*	*DP*	*Daimler*	*Thyssen*
BMW	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Adidas	0.32	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HEI.C	0.54	0.11	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DB	0.07	0.00	0.09	0.99	0.00	0.00	0.00	0.00	0.00	0.00
DTE	-0.07	-0.11	0.04	0.09	0.99	0.00	0.00	0.00	0.00	0.00
SZU	-0.08	-0.02	-0.03	0.04	0.13	0.99	0.00	0.00	0.00	0.00
Allianz	-0.12	-0.04	0.06	0.16	0.46	0.15	0.85	0.00	0.00	0.00
DP	0.00	-0.01	0.04	0.14	0.29	0.10	0.40	0.85	0.00	0.00
Daimler	-0.04	-0.04	0.03	0.18	0.33	0.15	0.46	0.22	0.75	0.00
Thyssen	0.03	-0.04	0.02	0.09	0.21	0.18	0.31	0.16	0.30	0.84

Source: Own calculation.

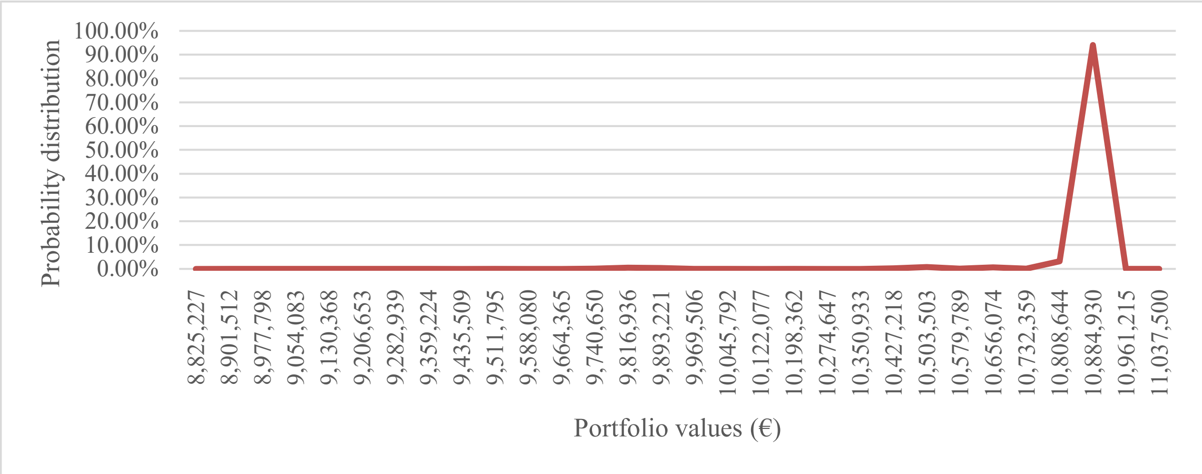
The Cholesky decomposition matrix can be obtained by means of Cholesky decomposition calculator. This matrix can be multiplied by random yields to get the correlated random variables under consideration of independence. The results of correlated random variables can be found in Annex 7.

Later, we can use IF founction in MS Excel to relate correlated random variables to credit rating with the help of the limits of the transition in each rating category and the results are presented in Annex 9. The breakpoints of limits of migration can be found in Annex 8. In this case, we can get the value of bonds of each scenario based on the assigned rating according to *Tab. 4.5*. After that, multiplying the value of bonds in each scenario and the number of bonds portfolio held, the total value of portfolio can be obtained by summing up the total value of each bond in one scenario. The value of each bonds and total values of portfolio of 30,000 scenarios are presented in Annex 10.

4.2.4 Calculation of credit risk

At last, we are trying to estimate the economic capital of portfolio. The values of portfolio of 30,000 scenarios can be divided into 30 equipartitions and the possibility distribution can be conducted with the equipartitions and the frequency of equipartitions by using FREQUENCY Founction of MS Excel. *Fig. 4.1* below represents the possibility distribution of the portfolio values and in this way we can also get the possibility distribution of default values which can be found in Annex 11.

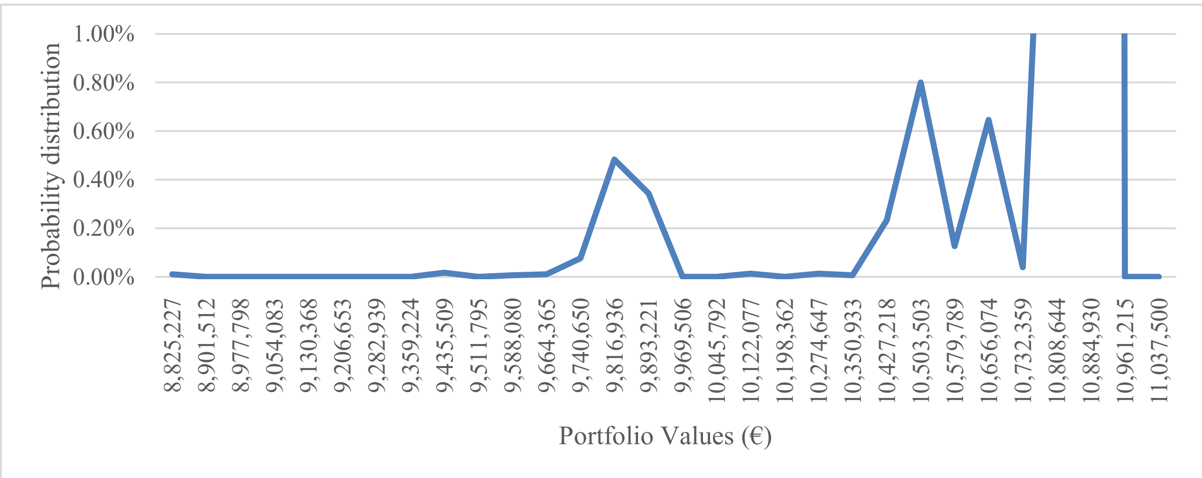
Fig. 4.1 Possibility distribution of the portfolio values



Source: Own calculation.

Illustrated in *Fig. 4.1,* we can see there is a left skewed possibility distribution with long-tail extentd to negative direction on horizontal axis which means the distribution of credit risk is asymmetric. Portfolio values occur most frequently around 10,701,019 *€* and the mean value is 10,664,569 *€*. The range of portfolio values from 10,624,734 *€* to 10,777,304 *€* contains 98.25% of the total area under the curve. Then let us enlarge the figure when the probability is from 0% to 1% and it can be showned as following *Fig. 4.2*.

Fig. 4.2 Possibility distribution of the portfolio values – adjust scale



Source: Own calculation.

Because of a negatively skewed possibility distribution, we can see that there is only a few scenarios whose portfolio values is quite lower accounting for low possibility. Illustrated in *Fig. 4.2* above, as an example of portfolio values ranging from 9,740,650 € to 9,893,221 €, they accounts for around 0.9% of all portfolio values which is significantly decline comparing to the possibility of most frequently range. Furthermore, the minimum possible potfolio value is 8,751,947 €.

Next, we are going to calculate the expected loss of each bonds and also portfolio. At now, the initial value of each bonds and the mean value of each bonds of 30,000 scenarios are kown. The expected loss is the difference between two parameters and the results can be shown as *Tab. 4.7* below.

Tab. 4.7 Results of the portfolio value (€)

	Value at initial rating	Expected value	Expected loss
BMW	1,115,212	1,114,914	-298
Adidas	1,089,900	1,089,722	-178
HeidelbergCement	1,131,337	1,131,024	-313
Deutsche Bank	1,066,128	1,063,720	-2,408
Deutsche Telekom	992,471	985,995	-6,476
Suedzucker	1,028,779	1,023,778	-5,001
Allianz	1,127,644	1,127,541	-103
Deutsche Post	1,134,657	1,134,396	-262
Daimler	997,126	997,260	135
Thyssenkrupp	1,151,483	1,151,321	-162
Portfolio	10,834,737	10,819,672	-15,065

Source: Own calculation.

The initial market price of portfolio is around 10,834,737 € and the expected portfolio value is 10,819,672 €. The difference between initial value of expected value is 15,065 € representing expected loss. Moreover, the expected loss accounts for 0.14% of total portfolio value and the reason might be the high rating of bonds. There is a positive expected loss from Daimler which means that this corporate face expected gain. The reason of this situation may be the relatively lower coupon rate with high rating. Deutsche Telekom contributes to the highest expected loss which is 6,475 € and accounts for 43% of total expected loss. One possible reason might be the risk associated with relatively lower credit rating. Later we calculate the risk as form of standard deviation and marginal risk of each bonds. The results summarize as *Tab. 4.8* below.

Tab. 4.8 Parameters of risk

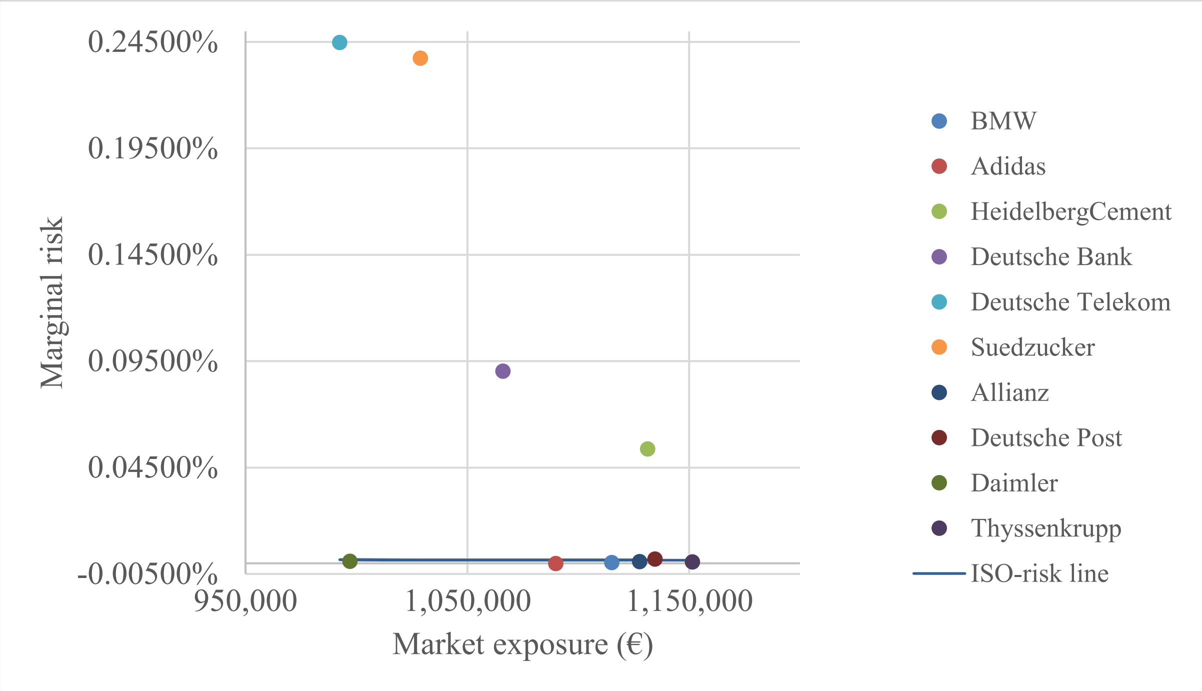
Bond	**Standard deviation**		**Marginal standard deviation**	
	%	**€**	**%**	**€**
BMW	0.122	1,355	0.000	-18
Adidas	0.076	829	0.000	-17
HeidelbergCement	0.051	532	0.054	11
Deutsche Bank	3.965	45,622	0.090	12,622
Deutsche Telekom	6.921	68,244	0.245	29,677
Suedzucker	4.916	50,655	0.237	15,686
Allianz	0.039	445	0.001	88
Deutsche Post	0.224	2,185	0.002	208
Daimler	0.148	1,474	0.001	100
Thyssenkrupp	0.101	1,166	0.001	64
Portfolio	**0.958**	**98,982**	-	-

Source: Own calculation.

Focus on three obligors with high expected loss, Deutsche Telekom, Suedzucker and Deutsche Bank have relatively higher credit risk by form of standard deviation which is 6.9%, 4.9% and 3.965%, respectively. The standard deviation of portfolio is around 0.958% and 98,982 € which is much lower than the sum of all the bonds because of diversificaton. As for maginal risk, three obligors with high expected loss also have higher marginal risk. There are several bonds whose marginal risk is very close to 0 which are BMW, Adidas, Allianz, Deustche Post, Daimler and also Thyssenkrupp representing high-quality of obligors.

Later, we would discuss the maiginal risk of ten bonds with the help of ISO-risk line. ISO-risk line combines all the points with the same absolute marginal risk. The fixed level of absolute marginal risk is estimated by absolute marginal risk of each bond multiplied by the market value and it is 16.61 €. The results can be shown as *Fig. 4.3* below.

Fig. 4.3 Marginal risk with ISO-risk line



Source: Own calculation.

Points around ISO-risk line can be referred to bonds with high market exposure but low probability. More specifically, we can see that those points around ISO-risk line represents six obligors with higher than A- rating and their exposure is much higher except Damiler. This can be explained that large exposures are typiccally allowed only if they have relatively small percentage risk levels. Points which fall above the ISO-risk line have greater absolute risk. The obligors with much higher absolute risk illustrated in *Fig. 4.3* includes HeidelbergCement, Deustche Bank, Suedzucker and Deustche Telekom.

At last, we are going to compare different portfolio value and VaR with different significance level. There are three different significance level used here which is 0.1%, 0.5% and 1% and we can see the results shown as *Tab. 4.9* below.

Tab. 4.9 Percentiles and corresponding value of the portfolio and losses

alpha	Portfolio value (€)	VaR (€)
0.1%	9,524,525	-1,310,212
0.5%	9,677,182	-1,157,556
1%	10,282,653	-552,084

Source: Own calculation.

At 99.9% confidence level, the portfolio value is minimized due to facing a maximun VaR which are 9,524,525 € and 1,310,212 €, respectively. And it is obvious that there is a sharp decline on VaR when siginificence level changing from 0.5% to 1 %. After that, it is possible to calculate economic capital representing a buffer against unexpected loss from credit risk according to *Equation (3.1)*. The results can be found in the following *Tab. 4.10*.

Tab. 4.10 Percentiles and corresponding economic capital

alpha	**Economic capital (€)**
0.1%	1,140,044
0.5%	987,390
1%	381,923

Source: Own calculation.

At 99.9% confidence level, the economic capital reaches at 1,140,044 €. It is 987,390 € and 381,923 € at 99.5% and 99% confidence level, respectively. We can see that there is also a sharp decline even there is a just slight change of significance level due to a negatively skewed possibility distribution.

4.3 Calculation of credit risk under Basel I, II and III

As we have discussed in previous chapter, there are some different regulations of capital requirements to cover unexpected losses from credit risk based on Basel I, II and III. Here in this subchapter, we would like to analyze different capital requirements of credit risk with the methods we have mentioned at Chapter 3.4.

4.3.1 Under Basel I

Under Basel I, the esitmation of capital requirement is based on *Equation (3.24)*. The first step is estimation of RWA by means of the risk weights of each issuers according to *Tab. 3.12*. Capital requirement can be estimated after the calculation of RWA. The results of RWA and capital requirement represents as *Tab. 4.11* below.

Tab. 4.11 Regulatory capital requirement under Basel I

	Rating	Nominal value (€)	w	RWA (€)	CR (€)
BMW	A+	1,000,000	100%	1,000,000	80,000
Adidas	AA-	1,000,000	100%	1,000,000	80,000
HeidelbergCement	BBB-	1,000,000	100%	1,000,000	80,000
Deutsche Bank	BBB+	1,000,000	20%	200,000	16,000
Deutsche Telekom	BBB+	1,000,000	100%	1,000,000	80,000
Suedzucker	BBB-	1,000,000	100%	1,000,000	80,000
Allianz	AA	1,000,000	100%	1,000,000	80,000
Deutsche Post	A-	1,000,000	100%	1,000,000	80,000
Daimler	A	1,000,000	100%	1,000,000	80,000
Thyssenkrupp	A-	1,000,000	100%	1,000,000	80,000
Total	-	-	-	**9,200,000**	**736,000**

Source: Own calculation.

As we mentioned as Chapter 3.4, it is not necessary to take an obligor's rating into consideration while only industries should be considered when assigned risk weights of assets. Thus, due to the fact that ten obligors excluding Deustche Bank are corporates, the assigned risk weights excluding Deustche Bank are 100% and the risk weight of Deustche Bank is 20%. As it shown in the last row of *Tab. 4.11*, the RWA of portfolio is 9,200,000 € in total and the regulatory capital requirement is 736,000 €.

4.3.2 Under Basel II

The calculation of capital requirement under Basel II is affected by the rating of each obligor. Different rating and different industry results in different risk weights according to *Tab. 3.14*. Moreover, there are several ways to calculate the capital requirement as we discussed in previous chapter and here we would use standard approach and foundatoin internal ratings-based approach. The standard approach is much similar with tha calculation of capital requirement under Basel I and the results summarize as the *Tab. 4.12* below.

Tab. 4.12 Regulatory capital requirements under Basel II - SA

Basel II - SA	Rating	Nominal value (€)	w	RWA (€)	CR (€)
BMW	A+	1,000,000	50%	500,000	40,000
Adidas	AA-	1,000,000	20%	200,000	16,000
HeidelbergCement	BBB-	1,000,000	100%	1,000,000	80,000
Deutsche Bank	BBB+	1,000,000	100%	1,000,000	80,000
Deutsche Telekom	BBB+	1,000,000	100%	1,000,000	80,000
Suedzucker	BBB-	1,000,000	100%	1,000,000	80,000
Allianz	AA	1,000,000	20%	200,000	16,000
Deutsche Post	A-	1,000,000	50%	500,000	40,000
Daimler	A	1,000,000	50%	500,000	40,000
Thyssenkrupp	A-	1,000,000	50%	500,000	40,000
Total				**6,400,000**	**512,000**

Source: Own calculation.

Illusstrated in *Tab. 4.12*, the change of risk weights of corporates with A- rating and higher than A- rating vary from issuer to issuer. The credit rating pays an important role when assigning risk weights. Generally speaking, the higher the credit rating, the lower the risk weights. Here in our example, risk weights vary from 20% to 100%. As anexample of highest rating obligor, Allianz, the risk weight decreases from 100% under Basel I to 20% under Basel II. Moreover, tie risk weight of Deutsche Bank increases from 20% to 100% because of relative lower rating class. The following step to calculate the capital requirement is as the same as the calculation under Basel I. The size of risky assets is 6.4 million euro and the difference between RWA under Basel I and under Basel II is 2.8 million euro. The capital requirement to cover credit risk under Basel II by standard approach is 512,000 € which decreases by around 30.43% from 736,000 € to 512,000 €. We can see that there is a significant decline after considering the rating of obligors.

Moreover, capital requirement can be estimated under Basel II by foundation internal ratings-based approach. *Tab. 4.13* below represents the results of the calculation.

Tab. 4.13 Regulatory capital requirements under Basel II - FIRB

Basel II - FIRB	Rating	RWA (€)	CR (€)
BMW	A+	100,317	8,025
Adidas	AA-	52,235	4,179
HeidelbergCement	BBB-	564,392	45,151
Deutsche Bank	BBB+	482,497	38,600
Deutsche Telekom	BBB+	482,497	38,600
Suedzucker	BBB-	562,577	45,006
Allianz	AA	39,314	3,145
Deutsche Post	A-	479,283	38,343
Daimler	A	335,879	26,870
Thyssenkrupp	A-	479,283	38,343
Total		3,578,273	286,262

Source: Own calculation.

By using foundation IRB approach, banks are allowed to use internal empirical model to estimate three parameters (PD, LGD and EAD) and further influence the capital requirement. Because of that, the risk weights would be much lower when obligors have lower probability of default. Comparing to standard approach, we can see that there is a large decline of both RWA and capital requirement. As an example of Allianz, it shows the highest change of capital requirement from 16,000 € to 3,145 €, decreased by 80%, because of the highest rating. Thus, by foundation IRB approach, the lower the probability of default, the lower capital required to cover the risk. Moreover, the RWA decreases from 6.4 million euro to around 3.6 million euro and the capital requirement by using different approach decreases from 512,000 € to 286,407 €, almost by 44%. It results from the significant decline of RWA and capital requirement of each bond.

4.3.3 Under Basel III

Here the standard approach and foundation IRB approach are used to estimate capital requirement as well under Basel III. The capital requirement of liquidity risk should be taken into account since it firstly introduced under Basel III. Thus, the minimum capital adequacy required under Basel III is 10.5% including munimum total capital and capital conservation buffer according to *Tab. 3.15*. The results of calculation of capital reqirement can be found in *Tab. 4.14*, by using standard approach.

Tab. 4.14 Regulatory capital requirements under Basel III – SA

Basel III - SA	Rating	Nominal value (€)	w	RWA (€)	CR (€)
BMW	A+	1,000,000	50%	500,000	52,500
Adidas	AA-	1,000,000	20%	200,000	21,000
HeidelbergCement	BBB-	1,000,000	100%	1,000,000	105,000
Deutsche Bank	BBB+	1,000,000	100%	1,000,000	105,000
Deutsche Telekom	BBB+	1,000,000	100%	1,000,000	105,000
Suedzucker	BBB-	1,000,000	100%	1,000,000	105,000
Allianz	AA	1,000,000	20%	200,000	21,000
Deutsche Post	A-	1,000,000	50%	500,000	52,500
Daimler	A	1,000,000	50%	500,000	52,500
Thyssenkrupp	A-	1,000,000	50%	500,000	52,500
Total				**6,400,000**	**672,000**

Source: Own calculation.

Comparing it with the calculation by standardard approach under Basel II, we can see the process is the same resulting in the same results of RWA which is 6.4 million euro. However, the change of capital requirement results from the change of minimum capital requirement. The capital requirement including total capital equirement and conservation buffer under Basel III is 672,000 € which is much higher than the results of standard approach under Basel II. The difference is 160,000 € which can be considered as conservation buffer. Comparing it with the results under Basel I, the value of RWA and capital requirement decreased by 30.43% and 8.7%, respectively.

Then the size of capital requirement is estimated by foundation IRB approach. The process is almost the same with the procesure under Basel II, but the minimum capital adequacy ratio changes from 8% to 10.5%. The summary results can be found in *Tab. 4.15* bleow.

Tab. 4.15 Regulatory capital requirement under Basel III – FIRB

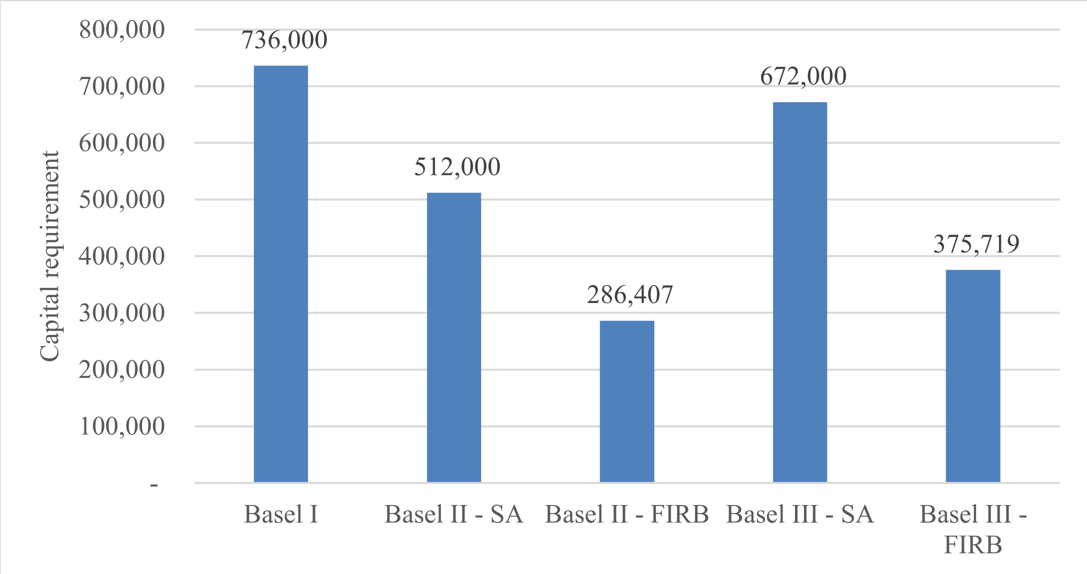
Basel III - FIRB	Rating	RWA	CR
BMW	A+	131,666	10,533
Adidas	AA-	68,558	5,485
HeidelbergCement	BBB-	740,765	59,261
Deutsche Bank	BBB+	633,277	50,662
Deutsche Telekom	BBB+	633,277	50,662
Suedzucker	BBB-	738,382	59,071
Allianz	AA	51,600	4,128
Deutsche Post	A-	629,058	50,325
Daimler	A	440,841	35,267
Thyssenkrupp	A-	629,058	50,325
Total		**4,696,484**	**375,719**

Source: Own calculation.

Illustrated as *Tab. 4.15* above, we can see that both RWA and CR increase comparing the results from FIRB approach under Basel III with the results from FIRB approach under Basel II. The absolute change of RWA and capital requirement is 1,116,395 € and 89,312 €, respectively. The changes are mainly because of the change of minimum capital adequacy ratio. When comparing the results from FIRB approach with the results from standard approach under Basel III, the capital requirement decreases from 672,000 € to 375,719 €. The absolute change is 296,281 € and the change expressed in percentage is around 44%.

In summary, all the results of calculation of capital requirement under different accords by each approach can be found in *Fig. 4.4* below.

Fig. 4.4 Regulatory capital requirement under Basel I, II and III



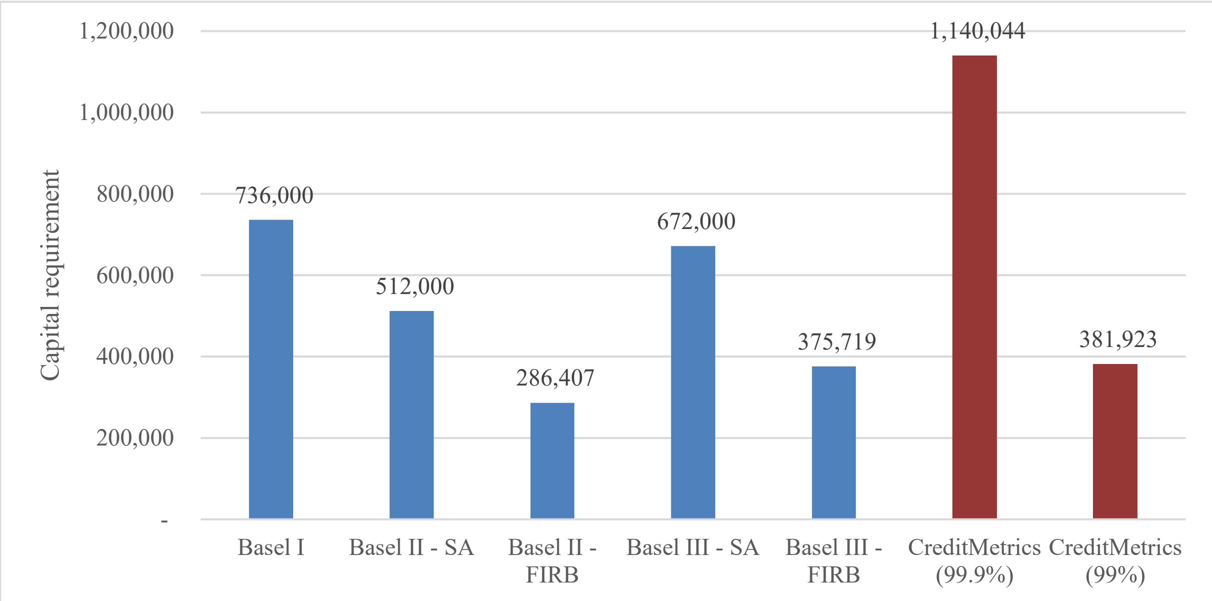
Source: Own calculation.

Illustrated in *Fig. 4.4*, there is a significant decrease of capital requirement when considering the rating of each obligor under Basel II. However, because of the introduction of liquidity risk and capital conservation buffer, the size of capital requirement increase a lot and the difference between capital requirement under Basel I and under Basel III – SA is just 64,000 €. If we consider the results under Basel I as an overvaluation of capital requirement, the results under Basel II should be undervaluation of capital requirement because banks behave not much well under financial stress in 2008 financial crisis. It might be a main reason of the release of Basel III Accord. On the other hand, we can see there is also a big jump on the results of capital requirement by using standard approach and foundation internal ratings-based approach both under Basel II and Basel III. The differences betwee standard spproach and foundation IRB approach under Basel II and Basel III are 225,593 € and 296,281 €, respectively. As we discussed above, the lower the probability of default, the lower the capital requirement and the risk weights would be never the same when the rating is not the same by using foundation IRB appraoch. However, the risk weights of different rating obligors (such as the risk weights of AA+, AA and AA-) might be the same when using standard appraoch. This might be the reason why the results of calculation by using foundation IRB approach is more significant when there is both SMEs and large corporates in the portfolio. Thus, by encouraging the usage of foundation internal ratings-based approach, it may distinguish the capital requirement of SMEs and large corporates.

4.4 Evaluation of results

In Chapter 4.2, the economic capital has been calculated by CreditMetrics^{TM} model by generating 30,000 random varibles at different confidence level. Then, we calculate the capital requirements by different approaches under three version of Basel Accord in Chapter 4.3. Due to the fact that the main idea of economic capital and capital requirement is to cover unexpected loss, at the end of this chapter, we would like to summary the results of each method. The results are presented graphically in *Fig. 4.5* below.

Fig. 4.5 Regulatory capital requiments under different methods



Source: Own calculation.

The main objectives of calculation of capital requirement under Basel Accord and calculation of economic capital under CreditMetrics^{TM} model are almost the same which is to compensate the unexpected loss. In other words, the results of these two methods can be compared together. As we discussed above, the capital reuirement under standard approach is higher due to same ways to estimate unexpected losses of large corporates and SMEs, and thus, foundation IRB approach is more significant when there are both corporates and SMEs in the portfolio.

Illustrated in *Fig. 4.5*, the value of economic capital under CreditMetrics^{TM} model with confidence level 99.9% is 1,140,044 € which is much higher than the capital requirement by using standar approach under Basel III. However, the result of economic capital with 99%

confidence level, which is 381,923 € is approximately the same with the result of capital requirement by using foundation IRB approach under Basel III.

The main reason to explain the different value estimated by two models is diversification. As we mentioned in previous chapter, the correlation of different obligors in one portfolio shoud be taken into account under CreditMetrics^{TM} model. Correlation should be considered as well in calculation of regulatory capital requirement under Basel Accords with the form of probability of default. We can see the huge difference between two correlation metrix considered in these two methods. Under CreditMetrics^{TM} model, the correlation metrix calculated by real share prices as representing in *Tab. 4.3* and the range is from -12% to 61%. However, the correlation (R), considered by foundation IRB approach, is calculated from probability of default which we can see in Annex 13 and it varies from 22% to 24%, a narrow interval. If correlation used by foundation IRB approach is considered highly independent, the portfolio with ten bonds is not perfectly diversified resulting in correlation has a wide interval.

5 Conclusion

With the development of contemporary banking system, it is significant that an appropriate risk management is implemented. After 2008 financial crisis, the measurement of financial risks, especially credit risk, became more and more important. The most common used methods implement by banking authorities is regulatory capital requirement based on Basel Accord and economic capital calculated under CreditMetrics^{TM} model. The main objective of this thesis is to estimate the capital requirement to cover unexpected losses from credit risk of ten selected bonds by different methods. It gives a possible way to compare the results from Basel Accords, including Basel I, Basel II and Basel III, and from CreditMetrics^{TM} model.

The main objective of this thesis is to estimate the economic capital of ten selected bonds portfolio under CreditMetrics^{TM} model and capital requirement for unexpected losses from credit risk under Basel Accord. It gives a possible way to compare the results from Basel Accords, including Basel I, Basel II and Basel III, and from CreditMetrics^{TM} model.

The whole thesis can be divided into five chapter. Chapter 2 and Chapter 3 constitute therotical part. Practical part can be found in Chapter 4 and Chapter 5 was structured on summary and conclusion of the results.

Therotical part mainly focused on different types of financial risks firstly and then description of credit risk management and models. Financial risks including credit risk, market risk, operational risk and liquidity risk were described in details with some examples. Later, several models for credit risk management were introduced and CreditMetrics^{TM} model was emphasized. At last, there was a desciption of different version of Basel Accords on capital adequacy.

In practical part, the example of a portfolio of ten selected bonds traded on Frankfurt Stock Exchange was used to economic capital by using CreditMetrics^{TM} model. Furthermore, the capital requirement to cover unexpected losses would be estimated by different approaches under different version of Basel Accords. The nominal value of whoe portfolio was 10 million euro and time horizon we selected was one year. And then, all the results would be analyzed and compared specifically.

The regulatory capital requirement under Basel Accords and economic capital under CreditMetrics^{TM} model can be found in practical part. The economic capital obtained under CreditMetrics^{TM} model with 99.9% confidence level was 1,140,044 € and the regulatory capital requirement by using standard approach under Basel III was 672,000 €. However, the economic capital determined under CreditMetrics^{TM} model with 99% confidence level, which was 381,923 €, was approximately similar to the regulatory capital requirement by foundation IRB approach under Basel III. The main reason to explain the different value estimated by two models was diversification. Our portfolio selected in practtical part can be considered as not perfectly diversified because of only ten bonds contained. Therefore, the correlation should be much higher than correlation of perfectly diversified porfolio. Under CreditMetrics^{TM} model, the correlation metrix was calculated by real share prices and the range was from -12% to 61%. However, the correlation (R), considered by foundation IRB approach, was calculated from probability of default which varied from 22% to 24%, a narrow interval.

Futhermore, the difference between regulatory capital requirement by using standard approach and foundation IRB approach can be also illustrated. The capital requirement by using foundation IRB, which was 375,719 €, was lower than capital requirement by using standard approach, which was 672,000 €. The main reason was that due to the fact that the risk weights assigned specifically to each rating categoties by using foundation IRB methods, the capital requirement decreased because high-quality obligors in the portfolio. Moreover, the changes of capital requirement under Basel II and Basel III can be also found in practical part. Capital requirement under Basel II was higher than capital requirement under Basel III by using both standard approach and foundation IRB approach because of a 2.5% conservation buffer.

The conservation buffer introduced in Basel III Accord represents that authorities prefer more capital requirement to compensate unexpected situation because of huge impact on banktrupcy of constitutions of banking system. In my opinion, if there is a futher version of Basel Accord, the regulatory capital requirement will be more and more restrict.

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List of Abbreviations

ACB	Agricultural Bank of China
BCBS	Basel Committee for Banking Supervision
BIS	Bank for Internationla Settlements
BOC	Bnak of China
CBIRC	China Banking and Insurance Regulatory Committee
CBRC	China Banking Regulatory Committee
CCB	China Construction Bank
CIRC	China Insurance Regulatory Committee
CR	Capital requirement
CRAs	Credit rating agencies
DD	Distance of default
EAD	Exposure at default
EL	Expected loss
EURINOR	Euro InterBank Offered Rate
FSE	Frankfurt Stock Exchange
IRS	Interest rate swap
LCR	Liquidity coverage ratio
LIBOR	London InterBnak Offered Rate
LGD	Loss given default
LLA	Loan loss allowance
LTD	Long-term debt
NPL	Nonperforming loans
NSFR	Net stable funding ratio
OECD	Organization for Economic Co-operation and Development
PD	probability of default
PRIBOR	Prague InterBank Offered Rate
RR	Recovery rate
RWA	Risk-weighted asset
STD	Shord-term debt
UL	Uxpected loss
VaR	Value at Risk

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Ostrava dated 21. 4. 2019

Yihan Gu 顾逸涵

Student's name and surname

List of Annexes

Annex 1: Probability matrix from Standard & Poor's (%)

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	85.03	6.72	1.52	0.87	0.22	0.43	0.00	0.00	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA+	1.09	74.86	15.03	2.73	0.82	0.82	0.55	0.55	0.00	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.22	1.20	78.98	8.50	4.14	1.31	0.54	0.22	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA-	0.08	0.08	4.56	74.98	12.26	2.73	1.24	0.17	0.08	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A+	0.00	0.07	0.63	5.51	73.97	10.89	2.58	0.49	0.35	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A	0.00	0.23	0.17	0.74	4.69	73.46	11.21	2.29	1.14	0.17	0.06	0.06	0.06	0.06	0.00	0.00	0.00	0.11
A-	0.05	0.00	0.16	0.16	0.98	7.22	76.11	7.93	1.48	0.82	0.16	0.05	0.11	0.00	0.00	0.00	0.00	0.05
BBB+	0.00	0.00	0.00	0.14	0.29	0.86	7.43	73.50	8.71	1.21	0.36	0.57	0.21	0.21	0.07	0.00	0.14	0.07
BBB	0.00	0.00	0.10	0.00	0.19	0.58	0.88	7.89	69.98	7.89	1.66	1.07	0.10	0.10	0.39	0.10	0.10	0.10
BBB-	0.00	0.00	0.16	0.00	0.16	0.64	0.48	1.43	8.90	67.25	6.52	2.70	0.79	0.32	0.32	0.00	0.32	0.32
BB+	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.60	0.90	11.64	58.81	8.06	2.39	1.79	0.30	0.00	0.30	0.00
BB	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.50	0.00	1.75	11.25	56.75	6.25	2.75	1.00	0.00	0.75	0.50
BB-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	8.89	59.01	12.84	4.20	0.49	0.25	1.48
B+	0.00	0.00	0.00	0.00	0.00	0.23	0.00	0.23	0.00	0.00	0.23	2.93	8.80	54.63	8.35	3.84	1.35	1.81
B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.38	0.38	1.51	12.08	45.66	8.30	4.53	4.15
B-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.00	0.00	0.00	0.00	1.27	6.33	49.37	15.82	10.13
CCC	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.15	3.46	9.20	25.29	37.93
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Annex 2: Shares prices from March 13rd, 2018 to March 12nd, 2019 (€)

Date	BMW	ADIDAS	HEI.C	DB	DTE	SZU	Allianz	DP	Daimler	Thyssenkrupp
2018/3/13	84.63	169.00	82.54	15.70	13.12	14.85	185.10	36.25	67.50	21.59
2018/3/14	84.17	187.90	82.06	15.70	13.15	15.00	188.58	36.75	68.24	21.81
2018/3/15	85.12	193.00	82.36	15.82	13.41	15.12	188.92	36.86	69.17	21.85
2018/3/16	85.86	194.10	82.28	15.54	13.27	14.75	185.78	36.33	68.53	21.66
2018/3/19	85.17	192.50	82.00	15.52	13.20	14.73	189.20	36.47	68.68	21.79
2018/3/20	85.99	194.65	82.06	14.79	13.14	14.86	188.70	36.20	68.63	22.16
2018/3/21	86.10	198.15	81.66	14.19	12.91	14.85	184.92	35.70	67.03	21.82
2018/3/22	85.33	197.00	79.76	13.75	12.95	14.86	180.36	35.04	65.77	21.16
2018/3/23	84.18	195.60	78.74	14.07	12.81	14.21	179.54	34.72	65.53	21.10
2018/3/26	85.21	193.00	77.98	13.77	12.98	13.20	182.34	35.16	66.47	21.54
2018/3/27	85.81	194.15	79.84	13.76	13.05	13.42	183.20	34.93	66.30	21.25
2018/3/28	85.56	192.95	79.70	13.98	13.25	13.79	183.40	35.52	68.97	21.19
2018/3/29	88.15	196.65	79.78	13.68	13.13	13.84	183.40	34.97	68.56	21.31
2018/4/3	88.68	195.30	79.14	13.72	13.12	13.44	182.72	34.89	68.11	21.01
2018/4/4	87.41	197.80	78.56	13.88	13.46	13.64	187.40	36.12	69.61	21.92
2018/4/5	88.70	204.20	80.96	14.32	13.45	13.64	186.66	35.98	65.33	21.64
2018/4/6	88.97	203.70	80.38	13.87	13.52	13.49	187.42	35.90	64.95	21.68
2018/4/9	88.34	202.60	81.08	14.01	13.56	13.67	188.48	36.52	65.74	22.16
2018/4/10	89.99	205.50	81.20	14.08	13.85	13.87	186.42	36.02	65.25	21.94
2018/4/11	89.87	206.50	79.40	14.19	13.87	13.98	188.64	36.34	65.47	22.13
2018/4/12	89.93	210.30	79.20	14.41	13.95	14.35	189.42	36.47	65.65	22.34
2018/4/13	91.33	210.60	79.72	14.38	13.87	14.34	189.48	36.48	65.31	22.18
2018/4/16	90.66	210.70	79.46	14.33	14.05	14.35	192.12	37.21	66.00	22.72
2018/4/17	91.22	213.20	80.18	14.38	14.00	14.00	192.12	37.76	65.23	23.04
2018/4/18	90.80	213.70	81.30	14.44	13.96	14.03	192.66	37.69	65.20	22.79
2018/4/19	90.88	210.50	81.82	14.46	14.05	14.00	193.54	37.71	65.12	22.71
2018/4/20	91.02	212.20	81.40	14.26	14.22	13.54	195.10	38.00	65.35	22.61
2018/4/23	91.20	211.70	81.70	14.25	14.12	13.44	194.86	37.85	65.16	22.69
2018/4/24	91.13	204.80	80.56	14.78	14.09	13.55	193.28	35.97	65.01	22.18
2018/4/25	89.27	202.30	80.56	14.60	14.34	13.72	195.14	35.89	65.03	21.89
2018/4/26	90.39	203.10	81.06	14.34	14.54	13.93	194.50	36.08	65.64	22.10
2018/4/27	91.50	204.50	81.00	13.89	14.50	13.79	196.46	36.10	65.50	21.60
2018/4/30	92.47	203.90	81.26	13.66	14.55	13.65	199.14	36.40	66.55	21.94
2018/5/2	93.09	207.20	82.44	13.59	14.37	13.75	197.32	36.41	66.33	21.65
2018/5/3	92.16	193.10	82.78	13.63	14.50	14.19	199.00	36.55	66.48	21.94
2018/5/4	91.56	192.70	84.00	13.63	14.57	14.45	201.00	37.15	66.83	22.40
2018/5/7	91.56	195.05	83.68	13.78	14.51	14.33	200.30	34.54	67.02	22.55
2018/5/8	91.94	193.60	83.20	13.78	14.30	14.03	198.54	34.13	67.10	22.82
2018/5/9	91.96	190.35	83.96	13.59	14.31	14.09	192.04	34.02	67.19	22.89
2018/5/10	92.36	190.60	81.08	13.78	14.27	14.08	191.54	34.43	67.05	23.21
2018/5/11	92.18	189.55	79.98	13.88	14.22	14.20	190.44	34.36	66.84	23.25

2018/5/14	91.83	189.10	79.34	13.86	14.16	14.28	192.26	33.99	66.90	21.75
2018/5/15	92.17	188.90	79.62	13.65	14.15	14.51	189.50	34.29	66.87	21.49
2018/5/16	92.16	193.90	79.88	13.29	14.14	14.57	191.80	34.20	67.80	21.53
2018/5/17	93.30	195.25	81.06	13.00	13.45	14.56	191.50	34.05	67.40	21.56
2018/5/18	88.57	195.05	80.46	12.99	13.45	14.56	191.50	34.05	67.40	21.56
2018/5/21	88.57	195.05	80.46	12.71	13.54	15.12	192.78	34.23	68.38	23.62
2018/5/22	90.83	195.50	81.10	12.71	13.52	14.92	188.88	33.85	67.08	23.08
2018/5/23	89.00	193.80	79.84	12.97	13.32	14.89	187.20	33.43	65.22	23.01
2018/5/24	87.50	193.35	79.00	12.88	13.37	14.99	187.54	33.45	65.07	23.08
2018/5/25	87.45	193.85	78.86	12.25	13.30	15.06	185.94	33.54	64.22	22.73
2018/5/28	87.22	195.60	77.90	12.05	13.16	14.85	180.30	33.05	63.19	22.84
2018/5/29	86.08	192.35	76.34	11.30	13.33	15.44	180.50	33.26	62.87	22.86
2018/5/30	86.20	196.85	75.82	11.57	13.22	15.34	176.38	32.47	61.68	22.57
2018/5/31	85.38	193.60	75.80	11.08	13.23	15.51	179.66	32.98	62.03	23.12
2018/6/1	85.88	193.15	77.00	11.04	13.33	15.43	180.90	33.22	62.16	22.97
2018/6/4	86.20	195.05	77.64	11.21	13.40	14.97	179.60	32.95	62.20	23.21
2018/6/5	86.59	196.40	77.80	11.03	13.45	15.22	178.72	32.87	62.35	23.72
2018/6/6	87.00	203.40	78.54	11.35	13.46	14.97	179.36	32.69	62.37	23.89
2018/6/7	86.63	198.20	78.40	11.51	13.34	14.99	178.58	31.17	61.96	23.36
2018/6/8	85.64	199.35	78.52	11.25	13.65	15.01	180.98	30.21	61.42	23.46
2018/6/11	85.38	198.40	78.76	11.43	13.79	14.93	181.16	29.46	62.31	23.33
2018/6/12	85.30	196.55	78.64	11.41	13.62	14.83	180.18	29.60	61.73	23.78
2018/6/13	84.68	194.90	77.02	11.37	13.85	14.82	183.06	30.36	63.19	23.69
2018/6/14	86.27	198.75	77.72	11.22	13.81	14.94	180.36	30.41	62.32	23.32
2018/6/15	85.74	199.85	76.90	11.10	13.61	14.91	178.22	30.23	61.73	22.90
2018/6/18	84.75	194.30	76.08	11.08	13.50	14.87	176.58	29.65	60.90	22.39
2018/6/19	84.06	189.40	75.00	11.17	13.47	14.69	175.92	29.82	60.45	22.07
2018/6/20	83.68	188.65	75.06	11.08	13.39	14.92	173.48	29.32	57.84	21.65
2018/6/21	81.22	190.15	73.30	10.89	13.54	14.74	175.72	29.23	57.66	21.48
2018/6/22	80.31	190.95	73.22	11.01	13.32	14.11	171.80	28.51	56.07	20.77
2018/6/25	78.85	187.00	71.74	10.89	13.20	14.07	172.04	28.05	55.63	20.22
2018/6/26	78.45	187.40	72.00	10.78	13.33	13.96	173.20	28.15	55.63	20.96
2018/6/27	78.61	189.10	73.12	10.38	13.26	13.60	172.70	27.90	55.23	20.60
2018/6/28	77.56	184.00	72.00	10.47	13.27	13.64	177.02	27.97	55.13	20.82
2018/6/29	77.63	186.95	72.08	10.62	13.29	13.51	174.38	27.72	55.58	20.62
2018/7/2	77.73	183.55	71.38	10.64	13.54	13.40	180.00	27.72	56.07	20.84
2018/7/3	77.70	185.00	71.74	10.70	13.74	13.21	179.04	27.53	56.38	20.94
2018/7/4	77.77	182.15	71.80	11.13	13.79	13.00	180.08	27.66	58.50	21.49
2018/7/5	80.66	181.85	72.64	11.46	13.84	12.72	179.62	27.59	58.04	22.01
2018/7/6	80.20	181.50	72.56	11.54	13.85	12.62	180.98	27.82	58.10	21.76
2018/7/9	80.21	181.85	72.62	11.43	13.89	12.59	181.80	28.29	58.35	21.55
2018/7/10	80.01	185.00	72.18	11.15	13.81	12.58	179.08	27.72	57.18	20.63
2018/7/11	78.87	183.05	70.94	11.20	13.87	11.93	179.18	27.65	57.16	20.90
2018/7/12	79.37	185.30	70.94	11.24	13.76	12.36	179.90	28.06	57.25	20.62

2018/7/13	79.62	187.75	70.96	12.14	13.73	12.60	180.08	28.20	57.08	20.60
2018/7/16	79.29	188.75	70.28	11.87	13.71	12.90	184.20	28.40	57.48	22.48
2018/7/17	80.01	185.75	70.88	12.08	13.78	12.62	184.72	28.74	58.26	22.87
2018/7/18	80.80	189.25	71.16	12.02	13.78	12.78	183.68	29.07	58.64	22.54
2018/7/19	80.75	189.50	71.22	12.03	13.75	12.26	181.30	28.93	57.26	21.85
2018/7/20	79.28	189.90	70.24	12.18	13.70	12.28	182.60	28.28	57.74	22.68
2018/7/23	79.93	189.45	69.30	12.28	13.61	12.30	184.58	28.68	59.17	23.19
2018/7/24	81.53	190.45	71.14	12.18	13.63	12.46	181.90	28.85	57.88	22.92
2018/7/25	79.86	188.60	70.38	12.20	13.87	12.38	184.80	29.70	59.51	23.45
2018/7/26	83.39	187.85	72.30	12.42	14.15	12.48	186.80	29.86	59.29	23.37
2018/7/27	82.91	189.20	72.92	12.79	14.18	12.48	188.08	29.89	59.35	22.92
2018/7/30	82.97	188.85	72.14	13.06	14.15	12.47	189.14	30.19	59.15	22.82
2018/7/31	82.69	189.15	72.62	12.95	14.17	12.43	188.40	30.10	58.24	22.67
2018/8/1	81.50	187.85	71.24	12.54	14.03	12.38	185.64	29.54	57.41	22.42
2018/8/2	81.18	184.95	69.96	12.50	14.24	12.11	187.40	29.53	58.16	21.87
2018/8/3	82.34	189.60	69.70	12.52	14.24	12.53	188.10	29.55	58.25	21.67
2018/8/6	83.51	188.20	71.00	12.62	14.27	12.44	188.58	30.67	58.89	21.56
2018/8/7	84.05	189.65	71.96	12.52	14.29	12.40	188.92	31.09	59.06	21.46
2018/8/8	84.48	190.55	72.48	12.40	14.28	12.36	189.64	31.31	59.04	21.09
2018/8/9	84.81	208.50	71.52	11.82	13.95	12.26	185.92	30.63	57.20	20.46
2018/8/10	83.58	205.70	70.10	11.45	14.02	12.36	186.66	30.80	57.23	20.30
2018/8/13	83.29	208.40	69.90	11.49	14.08	12.24	184.80	30.68	56.45	20.21
2018/8/14	82.45	209.30	70.10	11.19	13.92	12.03	184.00	30.16	55.05	19.43
2018/8/15	81.31	205.80	69.26	11.21	14.00	12.11	185.88	30.57	55.29	19.68
2018/8/16	81.80	208.70	70.34	11.20	13.95	12.12	184.24	30.75	54.92	19.33
2018/8/17	81.41	207.00	68.50	11.22	13.98	12.09	185.38	31.17	55.29	19.69
2018/8/20	81.75	209.50	69.36	11.54	13.99	12.08	185.54	31.24	55.87	19.90
2018/8/21	82.93	210.50	70.30	11.61	14.14	11.96	185.92	31.34	55.30	19.74
2018/8/22	82.29	210.40	70.08	11.30	14.17	12.03	185.54	31.40	54.41	19.67
2018/8/23	81.08	212.00	70.04	11.39	14.12	11.93	185.50	31.55	54.75	19.85
2018/8/24	81.29	213.70	70.60	11.73	14.18	11.96	187.92	31.88	56.11	20.20
2018/8/27	83.19	215.70	71.22	11.55	14.06	11.91	187.36	31.96	56.30	20.20
2018/8/28	84.39	215.70	70.66	11.66	14.11	12.00	187.84	31.83	56.67	20.14
2018/8/29	84.23	214.60	70.72	11.42	13.97	11.99	185.66	31.64	56.65	20.13
2018/8/30	84.60	215.20	69.82	11.26	13.91	11.84	183.64	31.41	55.70	19.91
2018/8/31	83.41	214.90	68.58	11.42	13.98	11.94	184.38	31.25	55.00	19.75
2018/9/3	82.79	216.00	68.10	11.47	13.74	12.03	182.10	31.00	54.20	19.87
2018/9/4	81.88	210.80	67.56	11.38	13.45	11.85	180.84	30.61	54.28	19.61
2018/9/5	81.25	207.00	67.38	11.13	13.44	11.59	180.54	30.81	54.32	19.37
2018/9/6	80.73	206.10	67.42	11.11	13.59	11.45	181.52	30.58	54.48	18.86
2018/9/7	81.07	207.20	66.00	11.24	13.65	11.77	182.34	30.70	54.50	19.17
2018/9/10	81.10	207.70	65.94	11.24	13.70	11.99	182.84	30.56	54.05	19.14
2018/9/11	80.76	206.60	65.60	11.36	13.66	12.23	185.00	30.98	54.53	19.32
2018/9/12	81.32	210.70	65.48	11.40	13.70	12.79	184.86	31.14	55.09	18.93

2018/9/13	82.47	209.70	65.46	11.51	13.75	12.68	185.90	31.11	55.54	18.97
2018/9/14	82.93	210.30	65.74	11.58	13.77	12.42	186.30	31.37	55.41	19.25
2018/9/17	82.54	208.20	65.54	11.95	13.90	12.30	186.70	31.78	55.77	19.68
2018/9/18	82.67	210.00	65.92	12.25	13.75	12.36	188.20	31.62	56.32	20.18
2018/9/19	83.54	209.20	67.60	12.34	13.84	12.00	190.20	31.74	57.21	20.32
2018/9/20	85.32	209.40	69.56	12.24	13.90	11.48	192.78	31.60	57.61	20.46
2018/9/21	85.77	210.90	69.32	12.15	13.80	11.20	192.18	31.49	56.13	20.27
2018/9/24	83.50	208.40	68.14	11.90	13.85	11.44	194.74	31.54	54.74	20.54
2018/9/25	79.00	210.70	68.16	11.81	13.97	11.34	195.86	31.08	54.83	20.07
2018/9/26	79.03	209.50	68.52	11.36	14.08	11.60	197.50	31.33	55.59	22.06
2018/9/27	79.00	211.50	67.90	11.25	13.89	11.45	192.00	30.71	54.35	21.74
2018/9/28	77.71	210.90	67.32	11.13	13.89	11.42	193.44	30.60	54.88	20.76
2018/10/1	78.14	213.80	67.80	11.32	13.86	11.82	192.46	30.46	56.00	20.90
2018/10/2	78.70	213.20	67.34	11.20	14.07	11.99	194.58	30.14	56.44	20.75
2018/10/4	78.26	209.00	67.88	11.12	14.02	12.12	193.04	29.66	55.61	20.60
2018/10/5	77.68	207.80	65.06	11.05	13.95	12.12	189.82	29.12	54.60	20.67
2018/10/8	76.87	206.30	63.72	11.03	13.91	12.56	190.96	29.10	54.42	20.86
2018/10/9	76.39	204.40	64.30	10.92	14.29	12.46	188.66	28.65	53.60	20.56
2018/10/10	75.35	195.15	62.94	10.83	14.00	12.93	182.28	28.19	52.70	19.92
2018/10/11	74.30	192.65	61.80	10.84	13.86	12.84	181.22	28.22	52.89	20.10
2018/10/12	74.49	192.60	61.10	11.03	14.15	12.50	182.06	28.38	52.92	19.99
2018/10/15	75.21	194.95	61.26	11.16	14.26	12.95	182.88	29.00	53.23	20.09
2018/10/16	75.75	200.00	62.42	11.26	14.40	13.37	184.34	28.95	52.76	20.33
2018/10/17	75.24	201.70	62.64	10.97	14.55	13.43	184.18	28.93	52.41	20.15
2018/10/18	75.13	198.75	57.26	10.92	14.66	13.70	185.20	28.89	51.39	19.34
2018/10/19	74.64	202.80	56.62	10.72	14.57	13.47	183.98	28.60	50.81	18.97
2018/10/22	74.44	200.90	56.56	10.70	14.42	13.33	181.42	27.97	50.48	18.42
2018/10/23	73.50	197.45	56.52	10.00	14.55	14.40	182.28	28.04	50.00	18.00
2018/10/24	72.69	201.00	55.38	10.00	14.44	14.13	183.14	28.48	51.35	18.25
2018/10/25	74.50	207.30	57.36	9.67	14.20	14.00	179.26	28.38	51.37	17.90
2018/10/26	75.02	203.40	57.46	9.64	14.22	14.20	181.18	28.68	52.43	18.28
2018/10/29	76.40	205.30	57.22	9.74	14.39	13.66	181.98	28.56	52.12	18.12
2018/10/30	76.34	202.10	58.10	9.77	14.50	13.65	184.44	27.95	52.36	18.58
2018/10/31	76.23	208.00	60.00	10.25	14.45	13.62	184.24	28.08	52.57	18.83
2018/11/1	76.79	204.00	60.12	10.52	14.39	13.47	184.92	28.05	53.14	19.17
2018/11/2	77.74	206.50	60.84	10.46	14.47	13.47	184.66	27.92	52.69	19.09
2018/11/5	77.23	207.00	59.82	10.44	14.51	13.09	183.98	28.88	52.27	19.08
2018/11/6	76.92	205.90	59.44	10.61	14.68	13.58	185.20	28.87	52.30	19.24
2018/11/7	74.26	198.60	60.30	10.34	14.66	13.26	187.02	28.62	51.30	19.06
2018/11/8	73.86	201.00	60.70	10.12	14.70	13.53	192.00	28.57	50.76	17.33
2018/11/9	73.36	203.90	60.10	9.73	14.69	13.37	191.00	28.36	50.41	16.80
2018/11/12	72.53	201.80	59.64	9.86	14.92	13.19	194.12	28.65	51.89	16.79
2018/11/13	73.90	206.60	61.06	9.87	14.97	13.27	190.80	28.56	52.29	16.66
2018/11/14	74.66	205.60	60.22	9.90	14.93	13.14	189.36	28.46	51.06	16.51

2018/11/15	74.20	205.50	59.86	9.81	15.13	13.08	189.24	28.44	50.49	16.41
2018/11/16	73.74	204.70	60.02	9.78	15.17	12.90	188.66	28.13	50.45	16.09
2018/11/19	74.18	200.80	59.22	9.23	15.19	12.37	184.96	27.93	49.78	15.75
2018/11/20	73.69	199.15	58.50	9.44	15.31	12.45	185.86	27.80	50.84	16.14
2018/11/21	74.32	204.40	59.28	9.25	15.11	12.42	183.46	27.50	50.42	16.20
2018/11/22	73.62	201.60	58.56	9.73	15.22	12.68	184.40	27.83	50.67	16.26
2018/11/23	73.49	202.70	58.84	9.84	15.38	12.99	189.52	28.49	51.94	16.59
2018/11/26	74.87	199.25	60.04	9.89	15.48	13.04	189.80	28.65	50.69	16.45
2018/11/27	73.87	199.15	59.82	9.42	15.38	13.02	189.32	28.48	50.62	16.67
2018/11/28	73.74	199.10	59.40	9.16	15.39	12.61	189.16	28.56	50.67	16.73
2018/11/29	72.85	197.50	59.46	9.39	15.49	12.57	186.50	28.12	49.70	16.50
2018/11/30	72.21	194.80	58.70	9.06	15.45	12.89	189.62	28.78	51.95	17.13
2018/12/3	75.66	197.70	59.50	8.90	15.43	12.76	188.38	27.99	50.45	16.91
2018/12/4	74.43	198.20	57.10	8.73	15.32	12.69	184.30	27.40	50.00	16.70
2018/12/5	74.16	196.45	57.36	8.38	15.03	12.45	176.02	26.43	46.91	15.98
2018/12/6	72.07	193.55	54.82	8.33	15.11	12.59	175.38	26.21	46.74	15.92
2018/12/7	71.71	195.45	55.50	9.03	14.99	12.29	173.78	25.52	45.47	15.48
2018/12/10	70.10	192.80	53.46	8.87	15.08	12.37	173.98	25.28	46.70	15.73
2018/12/11	71.38	197.50	54.28	8.80	15.19	12.31	176.40	25.24	47.42	15.80
2018/12/12	72.77	199.10	55.04	8.60	15.08	11.80	176.20	25.33	47.54	15.76
2018/12/13	74.18	200.00	54.18	8.69	15.11	11.99	176.84	25.09	47.37	15.61
2018/12/14	74.27	195.50	54.40	8.37	15.22	11.70	175.28	24.84	47.42	15.62
2018/12/17	74.00	186.85	53.92	8.06	15.14	11.66	174.76	25.27	47.10	15.79
2018/12/18	73.47	186.20	54.00	7.87	15.38	11.79	175.94	24.22	47.22	16.08
2018/12/19	73.45	187.05	54.20	7.91	15.13	11.42	174.96	23.72	46.51	15.50
2018/12/20	71.84	183.75	53.24	8.16	15.01	11.39	175.02	24.12	46.81	15.28
2018/12/21	71.93	184.75	53.88	7.81	14.59	10.99	172.16	23.73	45.27	14.81
2018/12/27	69.86	180.10	52.56	8.05	14.82	11.30	175.14	23.91	45.91	14.98
2018/12/28	70.70	182.40	53.38	8.15	14.82	11.10	175.00	23.99	45.25	14.91
2019/1/2	69.74	184.40	53.28	8.20	14.93	11.10	173.16	23.54	44.78	14.61
2019/1/3	69.05	183.95	52.50	8.09	15.05	11.36	177.36	24.40	47.07	15.37
2019/1/4	71.71	191.05	55.02	8.52	15.07	11.26	175.92	24.58	47.16	15.56
2019/1/7	72.12	189.70	54.80	8.70	14.87	11.69	176.10	24.89	47.50	15.84
2019/1/8	72.21	194.30	55.36	8.74	14.77	11.75	177.14	25.36	48.92	16.16
2019/1/9	72.95	193.85	56.52	8.56	14.78	12.68	178.52	25.05	49.29	16.27
2019/1/10	72.53	195.50	56.58	8.61	14.74	13.03	178.96	24.98	48.90	16.18
2019/1/11	71.83	197.00	56.50	8.56	14.71	13.30	179.22	24.91	49.28	16.11
2019/1/14	71.59	197.30	56.66	8.58	14.69	13.39	179.24	24.42	49.44	16.11
2019/1/15	71.62	199.25	57.80	8.63	14.68	13.23	180.40	25.03	49.85	15.86
2019/1/16	71.53	198.10	58.56	9.29	14.59	13.90	180.24	25.02	48.94	15.70
2019/1/17	71.26	200.00	58.80	8.92	14.89	14.04	184.00	25.77	50.89	15.98
2019/1/18	73.34	204.00	60.14	9.14	14.53	14.14	184.40	25.63	50.72	15.89
2019/1/21	73.03	206.00	59.96	8.85	14.54	13.84	183.32	25.61	50.57	15.53
2019/1/22	72.73	205.80	60.70	9.05	14.46	13.64	183.28	25.83	50.28	15.26

2019/1/23	72.23	207.30	59.98	8.86	14.19	13.69	185.00	25.80	51.09	15.29
2019/1/24	73.12	202.30	60.46	9.24	14.10	13.80	185.92	26.31	52.57	15.43
2019/1/25	74.35	204.00	61.52	9.27	14.20	13.60	184.44	26.26	52.19	15.54
2019/1/28	73.98	203.10	60.68	9.23	14.29	14.19	185.64	26.09	52.00	15.40
2019/1/29	73.85	204.30	60.70	9.29	14.24	14.15	185.50	25.69	51.77	15.41
2019/1/30	73.33	206.20	60.82	8.88	14.19	14.18	184.92	25.76	51.66	15.47
2019/1/31	73.46	207.70	60.36	8.85	14.21	14.46	186.16	25.79	53.03	15.89
2019/2/1	74.14	198.85	61.26	8.80	14.16	14.44	185.86	25.62	52.17	15.56
2019/2/4	73.24	196.75	61.12	8.90	14.49	14.30	187.60	26.13	52.91	15.52
2019/2/5	73.61	202.80	61.80	8.95	14.34	14.10	187.12	26.15	51.95	15.90
2019/2/6	73.65	202.00	61.62	8.36	14.19	13.64	184.52	25.39	49.22	15.09
2019/2/7	71.22	199.50	59.06	8.19	14.14	13.52	182.72	25.26	48.01	14.42
2019/2/8	69.53	198.35	58.38	8.32	14.16	13.81	184.04	25.88	48.16	14.63
2019/2/11	69.41	199.50	58.88	8.54	14.16	13.62	185.28	26.07	49.38	14.34
2019/2/12	69.91	201.10	58.96	8.54	14.17	13.40	185.66	26.25	50.22	13.69
2019/2/13	70.39	198.65	60.06	8.34	14.15	13.34	184.20	26.00	49.30	13.30
2019/2/14	69.35	197.00	60.00	8.80	14.26	13.44	190.14	26.75	50.61	13.26
2019/2/15	70.77	199.15	61.82	8.61	14.38	13.42	189.52	26.11	50.40	13.04
2019/2/18	70.59	198.55	62.18	8.74	14.48	13.45	191.04	26.39	50.46	13.16
2019/2/19	71.04	201.80	64.40	8.66	14.58	13.49	192.00	26.71	51.76	13.28
2019/2/20	72.51	202.10	65.06	8.65	14.64	13.62	193.20	26.84	52.11	13.19
2019/2/21	73.12	202.80	64.62	8.77	14.68	13.64	194.06	26.75	52.20	13.41
2019/2/22	73.13	204.10	64.82	8.92	14.58	13.47	194.14	26.57	53.44	13.71
2019/2/25	73.70	210.30	64.92	9.11	14.50	13.07	195.24	27.05	53.15	13.54
2019/2/26	74.00	213.90	64.90	9.24	14.48	13.00	194.48	27.12	52.88	13.32
2019/2/27	74.24	213.10	64.90	9.29	14.49	12.86	195.60	27.32	52.66	13.15
2019/2/28	74.31	213.60	64.64	9.20	14.53	13.09	197.34	27.30	53.28	13.24
2019/3/1	74.75	215.90	65.14	9.16	14.59	12.94	197.70	27.42	53.48	13.22
2019/3/4	74.67	215.80	64.96	9.17	14.69	12.90	198.14	27.40	52.91	13.16
2019/3/5	75.06	216.70	65.04	8.66	14.69	12.98	198.46	27.10	52.20	13.06
2019/3/6	74.53	217.60	65.00	8.64	14.87	12.56	197.26	27.45	50.44	12.61
2019/3/7	73.09	215.60	63.84	9.12	14.87	12.65	195.88	27.65	50.08	12.29
2019/3/8	72.16	210.60	63.18	8.86	14.97	12.61	196.80	28.07	50.75	12.30
2019/3/11	73.18	212.60	63.92	9.06	15.01	12.56	196.66	28.45	50.56	12.35
2019/3/12	73.10	209.80	64.20	8.90	15.11	12.58	198.44	28.81	50.88	12.45

Annex 3: Covariance matrix

	BMW	***ADIDAS***	***HEI.C***	***DB***	***DTE***	***SZU***	***Allianz***	***DP***	***Daimler***	***Thyssenkrupp***
BMW	0.00017	0.00007	0.00010	0.00002	-0.00001	-0.00002	-0.00002	0.00000	-0.00001	0.00001
ADIDAS	0.00007	0.00030	0.00007	0.00001	-0.00002	-0.00002	-0.00002	0.00000	-0.00001	-0.00001
HEI.C	0.00010	0.00007	0.00022	0.00004	0.00000	-0.00002	0.00000	0.00001	0.00000	0.00001
DB	0.00002	0.00001	0.00004	0.00054	0.00002	0.00002	0.00004	0.00005	0.00007	0.00005
DTE	-0.00001	-0.00002	0.00000	0.00002	0.00010	0.00003	0.00005	0.00004	0.00005	0.00005
SZU	-0.00002	-0.00002	-0.00002	0.00002	0.00003	0.00040	0.00005	0.00004	0.00006	0.00009
Allianz	-0.00002	-0.00002	0.00000	0.00004	0.00005	0.00005	0.00013	0.00008	0.00011	0.00010
DP	0.00000	0.00000	0.00001	0.00005	0.00004	0.00004	0.00008	0.00021	0.00012	0.00011
Daimler	-0.00001	-0.00001	0.00000	0.00007	0.00005	0.00006	0.00011	0.00012	0.00025	0.00018
Thyssenkrupp	0.00001	-0.00001	0.00001	0.00005	0.00005	0.00009	0.00010	0.00011	0.00018	0.00046

Annex 4: Yield curves derived from the annual transition matrix (%)

1st year: 2019

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	72.38	10.76	3.54	1.72	0.60	0.81	0.11	0.07	0.35	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA+	1.78	56.30	23.27	5.43	2.23	1.63	1.10	0.92	0.10	0.42	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.38	1.87	62.98	13.36	7.45	2.73	1.22	0.45	0.07	0.19	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
AA-	0.14	0.19	7.12	57.31	18.59	5.54	2.54	0.49	0.24	0.28	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
A+	0.01	0.14	1.25	8.35	55.96	16.41	5.20	1.22	0.72	0.18	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.01
A	0.01	0.35	0.38	1.40	7.13	55.34	17.08	4.37	2.03	0.46	0.14	0.13	0.11	0.09	0.01	0.00	0.01	0.20
A-	0.08	0.02	0.28	0.37	1.86	10.99	59.37	12.16	3.01	1.42	0.33	0.18	0.18	0.04	0.02	0.00	0.02	0.11
BBB+	0.00	0.00	0.03	0.24	0.58	1.89	11.31	55.34	12.73	2.51	0.78	0.93	0.37	0.34	0.16	0.04	0.16	0.20
BBB	0.00	0.00	0.17	0.04	0.35	1.04	1.99	11.53	50.40	11.15	2.80	1.76	0.33	0.29	0.52	0.17	0.18	0.27
BBB-	0.00	0.00	0.25	0.03	0.29	1.02	0.98	2.82	12.41	46.76	8.68	4.06	1.37	0.74	0.51	0.08	0.37	0.71
BB+	0.00	0.00	0.02	0.00	0.03	0.11	0.53	1.10	2.26	14.90	36.28	9.91	3.58	2.64	0.70	0.13	0.40	0.27
BB	0.00	0.00	0.00	0.00	0.01	0.04	0.41	0.77	0.32	3.51	13.14	33.80	7.78	4.20	1.58	0.29	0.75	1.26
BB-	0.00	0.00	0.00	0.00	0.00	0.03	0.03	0.10	0.35	0.54	1.36	10.71	36.58	15.36	5.60	1.40	0.72	2.95
B+	0.00	0.00	0.00	0.00	0.01	0.30	0.05	0.32	0.07	0.13	0.65	4.10	10.32	32.13	9.06	4.85	2.11	4.19
B	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.04	0.09	0.48	0.50	0.92	2.68	12.48	22.61	8.78	4.70	8.85
B-	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.05	0.75	0.07	0.04	0.07	0.21	2.27	6.67	26.40	12.12	21.42
CCC	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.01	0.02	0.05	0.15	1.45	3.13	7.20	8.02	48.62
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

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10th year: 2028

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	17.63	9.04	11.12	6.44	5.11	3.78	2.54	1.30	0.81	0.44	0.12	0.08	0.04	0.03	0.02	0.01	0.01	0.07
AA+	1.75	5.76	15.60	10.80	9.89	7.13	5.31	2.64	1.36	0.77	0.23	0.17	0.09	0.07	0.04	0.01	0.02	0.15
AA	0.57	1.43	11.94	11.17	11.66	8.85	6.52	2.94	1.50	0.74	0.23	0.16	0.09	0.07	0.03	0.01	0.02	0.16
AA-	0.26	0.61	5.80	10.61	13.12	11.34	8.91	4.11	2.16	1.00	0.33	0.24	0.13	0.10	0.05	0.02	0.02	0.25
A+	0.11	0.35	2.61	5.90	10.61	12.15	10.96	5.57	2.99	1.34	0.46	0.34	0.19	0.15	0.08	0.03	0.04	0.40
A	0.09	0.28	1.25	2.63	5.76	11.18	13.30	7.99	4.48	2.11	0.78	0.59	0.33	0.26	0.14	0.06	0.06	0.94
A-	0.11	0.16	0.74	1.49	3.61	8.63	14.05	9.99	5.86	2.91	1.12	0.84	0.46	0.37	0.20	0.09	0.09	0.99
BBB+	0.04	0.06	0.37	0.72	1.79	4.54	8.95	10.20	7.42	4.00	1.66	1.25	0.67	0.57	0.32	0.15	0.14	1.70
BBB	0.02	0.04	0.28	0.40	0.98	2.39	4.64	6.73	7.14	4.94	2.22	1.61	0.88	0.75	0.41	0.20	0.17	2.31
BBB-	0.01	0.03	0.26	0.30	0.69	1.58	2.75	4.09	5.23	5.08	2.60	1.94	1.18	1.02	0.52	0.27	0.21	3.54
BB+	0.00	0.01	0.10	0.11	0.28	0.73	1.33	2.07	2.89	3.59	2.33	1.94	1.37	1.25	0.61	0.34	0.23	3.66
BB	0.00	0.01	0.05	0.05	0.14	0.39	0.73	1.10	1.48	2.16	1.76	1.74	1.45	1.39	0.68	0.41	0.25	5.96
BB-	0.00	0.00	0.02	0.02	0.06	0.18	0.30	0.47	0.64	1.05	1.15	1.61	1.86	1.99	1.03	0.71	0.39	12.04
B+	0.00	0.00	0.01	0.02	0.07	0.18	0.26	0.34	0.39	0.59	0.66	1.01	1.26	1.47	0.80	0.62	0.33	15.34
B	0.00	0.00	0.01	0.01	0.02	0.08	0.11	0.18	0.23	0.32	0.34	0.53	0.69	0.89	0.53	0.46	0.25	22.09
B-	0.00	0.00	0.00	0.01	0.02	0.05	0.08	0.15	0.19	0.16	0.13	0.20	0.27	0.41	0.29	0.32	0.17	40.74
CCC	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.05	0.05	0.05	0.08	0.12	0.17	0.12	0.12	0.07	56.98
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Annex 5: Forward yield curves from 2019 to 2028

	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
AAA	-0.24%	-0.13%	0.04%	0.30%	0.45%	0.68%	0.89%	1.11%	1.28%	1.40%
AA+	-0.24%	-0.13%	0.04%	0.30%	0.46%	0.68%	0.89%	1.12%	1.29%	1.41%
AA	-0.24%	-0.13%	0.04%	0.30%	0.46%	0.68%	0.89%	1.12%	1.29%	1.41%
AA-	-0.24%	-0.13%	0.05%	0.31%	0.46%	0.68%	0.89%	1.12%	1.29%	1.41%
A+	-0.23%	-0.12%	0.05%	0.31%	0.47%	0.69%	0.90%	1.13%	1.30%	1.42%
A	-0.14%	-0.06%	0.10%	0.36%	0.50%	0.72%	0.93%	1.16%	1.33%	1.45%
A-	-0.19%	-0.09%	0.08%	0.34%	0.50%	0.72%	0.93%	1.16%	1.33%	1.45%
BBB+	-0.14%	-0.05%	0.13%	0.39%	0.54%	0.76%	0.97%	1.20%	1.37%	1.49%
BBB	-0.11%	-0.01%	0.16%	0.42%	0.58%	0.80%	1.01%	1.23%	1.40%	1.52%
BBB-	0.11%	0.14%	0.28%	0.53%	0.67%	0.88%	1.09%	1.31%	1.47%	1.58%
BB+	-0.11%	0.03%	0.22%	0.50%	0.65%	0.88%	1.09%	1.31%	1.48%	1.59%
BB	0.38%	0.37%	0.50%	0.73%	0.86%	1.06%	1.25%	1.46%	1.61%	1.72%
BB-	1.24%	1.00%	1.06%	1.26%	1.36%	1.53%	1.69%	1.87%	2.00%	2.07%
B+	1.89%	1.58%	1.58%	1.72%	1.76%	1.87%	1.99%	2.13%	2.23%	2.28%
B	4.49%	3.36%	2.97%	2.84%	2.69%	2.66%	2.67%	2.72%	2.75%	2.74%
B-	13.05%	9.38%	7.71%	6.70%	5.90%	5.39%	5.03%	4.80%	4.59%	4.40%
CCC	45.89%	23.98%	16.59%	12.94%	10.67%	9.25%	8.27%	7.59%	7.05%	6.60%

Annex 6: Random variables

	$\tilde{\varepsilon}_{BMW}$	$\tilde{\varepsilon}_{\mathrm{ADIDAS}}$	$\tilde{\varepsilon}_{\mathrm{HELC}}$	$\tilde{\varepsilon}_{\mathrm{DB}}$	$\tilde{\varepsilon}_{\mathrm{DTE}}$	$\tilde{\varepsilon}_{\mathrm{SZU}}$	$\tilde{\varepsilon}_{\mathrm{Allianz}}$	$\tilde{\varepsilon}_{\mathrm{DP}}$	$\tilde{\varepsilon}_{\mathrm{Daimler}}$	$\tilde{\varepsilon}_{\mathrm{Thyssenkrupp}}$
1	-0.5357	0.4344	0.4707	0.3581	-1.1929	-1.3648	-1.2641	-0.2512	0.2085	1.6509
2	-0.8698	0.4513	-0.3126	-2.1988	0.7949	-1.0095	-0.5941	1.0459	1.6455	0.7799
3	0.2510	0.3470	-0.5738	1.3683	-0.6302	-1.3419	0.4307	-0.7112	0.6096	0.8464
4	1.8554	0.7385	0.2376	0.0659	0.6498	1.1630	-1.0068	0.4038	-0.1132	0.6189
5	0.5085	0.0047	-0.6174	-1.3837	0.6798	0.0973	1.3372	0.7553	-0.7533	-0.5484
6	0.4096	1.2902	0.0486	2.4956	2.4350	-2.2109	0.1252	0.4890	1.2892	-0.5864
7	1.2284	1.4251	-0.1855	-2.6346	0.9532	0.5389	-0.1579	-0.1878	0.3269	1.0443
8	-0.8941	-0.9513	0.6786	-0.2136	0.4290	-0.5132	-0.3446	-0.4554	-0.4848	-0.3722
9	1.1316	1.2984	-1.4507	-0.7007	0.7160	-0.1943	-0.3446	-0.0192	0.2250	-0.3949
10	2.1903	0.4410	-0.5333	-0.2211	-0.0748	-0.2187	0.6624	-0.7488	1.6312	2.2603
11	0.4309	0.0258	-1.9127	0.7356	1.0243	0.4106	0.6153	0.1214	0.9601	0.7736
12	-2.0158	-1.3986	1.3292	1.0328	1.6806	0.7488	0.4398	-1.0227	1.7719	1.3941
13	1.4687	0.4543	-0.4017	-0.2769	-0.6269	0.1767	-0.1271	-0.8491	2.3968	-0.2634
14	0.2900	0.8446	0.9005	-0.6663	-0.3302	0.2794	2.1016	-0.0431	1.8126	0.1504
15	-0.9530	-1.2966	0.5575	-0.9570	1.5423	0.1890	-0.7872	2.6960	2.2127	-0.8578
16	3.2340	1.2294	-1.9913	0.1601	0.1416	1.6402	0.0465	-0.0578	-0.3314	0.2260
17	-1.1679	0.8333	1.4629	0.3335	-1.9502	-0.0492	-0.6466	0.8718	-1.5304	-0.2111
18	-1.9247	0.2872	0.8692	-0.6829	0.5728	1.1130	-0.0745	0.8642	1.4911	1.5950
19	1.1050	-0.4070	-0.3819	-1.7635	-0.7347	-0.9047	-0.3603	-0.2887	-0.8943	2.6297
20	1.2530	1.0400	0.2443	-0.5303	0.1046	0.7573	0.3881	-0.3148	-0.2681	0.0243
21	1.2943	-0.6072	0.6183	0.0349	0.1911	2.1281	-1.9122	-0.1177	1.3913	0.1258
22	-0.4355	-0.9108	1.1048	-0.9694	0.1181	-1.9606	0.9115	2.1778	0.3742	-0.6033
23	0.8486	0.7392	-0.4068	-0.5938	0.7867	-1.6978	-0.6398	-1.5143	-0.1148	1.3102
24	-1.1060	1.2325	-0.7759	-1.0704	-1.6453	0.2525	0.5629	0.9938	-0.5470	-0.7159
25	0.2581	-1.3127	0.8394	0.8457	1.6602	-1.9991	0.7467	0.1037	1.7335	0.5377
26	-0.0879	0.5607	-0.4981	-0.7549	0.4864	-0.8728	1.7335	-0.0825	-1.6453	1.0782
27	0.2629	0.8824	-0.0174	-0.9913	0.3539	-0.2488	-1.2326	0.6393	1.1023	-1.1145
28	0.8866	0.1508	0.6708	-2.3376	1.1579	-0.1747	-0.0119	-1.6660	1.1236	-0.4909
29	0.6072	-0.3700	-2.0140	-0.4164	-1.5950	0.1694	0.1502	1.6716	-0.5271	-0.5975
30	-1.9204	-1.6359	0.4499	0.3103	-1.0780	0.1466	0.5012	0.5417	-0.8708	-0.4383
31	-1.3950	1.7325	0.1196	0.8208	-0.0200	-1.0537	-0.6413	-0.7478	-0.6006	-1.0003
32	-0.4223	-1.3320	-1.2129	-0.7235	0.4636	-0.4775	0.8221	0.7853	-0.6792	1.6272
33	1.2384	-0.0998	-0.0774	-1.5348	0.3712	0.3712	-0.2452	-0.3737	0.9056	2.3927
34	0.6399	-0.5064	0.9179	-0.8579	-1.2085	-0.4039	1.1297	0.3469	0.8615	1.1736
35	-0.0707	-1.6124	-0.4457	0.4312	-2.7839	1.3986	1.6682	-0.4044	0.4874	0.3687
Etc.	…	…	…	…	…	…	…	…	…	…

Annex 7: Correlated random variables

	$\tilde{\varepsilon}_{BMW}$	$\tilde{\varepsilon}_{\mathbf{ADIDAS}}$	$\tilde{\varepsilon}_{\mathbf{HEI.C}}$	$\tilde{\varepsilon}_{\mathbf{DB}}$	$\tilde{\varepsilon}_{\mathbf{DTE}}$	$\tilde{\varepsilon}_{\mathbf{SZU}}$	$\tilde{\varepsilon}_{\mathbf{Allianz}}$	$\tilde{\varepsilon}_{\mathbf{DP}}$	$\tilde{\varepsilon}_{\mathbf{Daimler}}$	$\tilde{\varepsilon}_{\mathbf{Thyssenkrupp}}$
1	0.2560	0.6068	0.3752	0.1541	-1.5936	-1.2350	-0.5575	0.0990	0.6466	1.3802
2	-0.9919	0.2328	-0.3317	-1.7319	1.3897	-0.5943	0.9236	1.3821	1.4696	0.6520
3	0.2471	0.2951	-0.3040	1.4075	-0.4206	-1.0835	0.6286	-0.3337	0.7098	0.7076
4	2.2229	0.6516	0.1576	0.1073	0.5315	1.1288	-0.5499	0.4187	0.0984	0.5174
5	-0.1079	-0.1576	-0.5371	-1.1854	1.1530	0.1653	0.9161	0.3874	-0.7296	-0.4585
6	0.9542	0.9766	0.4787	2.8640	2.6280	-2.0274	0.7155	0.6080	0.7962	-0.4902
7	1.3253	1.1602	-0.3624	-2.4036	1.2071	0.7281	0.2706	0.0813	0.5557	0.8731
8	-0.7811	-0.8077	0.5159	-0.4382	-0.1679	-0.7459	-0.8168	-0.5556	-0.4752	-0.3112
9	0.7047	1.0138	-1.2710	-0.6953	0.5109	-0.2852	-0.3200	-0.0302	0.0522	-0.3301
10	1.9687	0.1896	-0.3544	0.2768	0.9963	0.4679	1.7282	0.0894	1.8979	1.8896
11	-0.7267	-0.4081	-1.4177	1.1998	1.8591	0.7973	1.2590	0.4414	0.9520	0.6467
12	-1.9336	-1.5122	1.3133	1.5870	2.5379	1.2240	1.2221	-0.2525	1.7468	1.1655
13	1.3073	0.3728	-0.3722	-0.0471	-0.1604	0.3806	0.5791	-0.2336	1.7254	-0.2202
14	0.6723	0.7569	0.8594	-0.0085	1.2952	0.8949	2.6530	0.3900	1.4086	0.1257
15	-1.2711	-1.3980	0.5388	-0.2345	2.5120	0.5104	1.1699	2.6488	1.4106	-0.7171
16	2.4362	0.8972	-1.6994	0.2017	0.2890	1.6117	-0.0662	-0.0862	-0.1823	0.1889
17	0.1696	1.2619	1.1309	-0.1206	-2.5295	-0.3287	-0.9742	0.3687	-1.2142	-0.1765
18	-1.5531	0.1485	0.7634	-0.0491	1.7431	1.6879	1.4763	1.3247	1.5951	1.3334
19	0.9294	-0.3851	-0.4823	-1.8657	-0.8399	-0.6318	-0.0090	-0.0197	0.1070	2.1985
20	1.5728	0.9823	0.1412	-0.5147	0.2036	0.7400	0.0863	-0.3237	-0.1946	0.0203
21	1.4146	-0.5520	0.3787	0.0912	0.0296	2.0246	-0.9853	0.2289	1.0843	0.1051
22	-0.1811	-0.7702	1.0473	-0.5792	0.9122	-1.6297	1.6312	1.8405	0.1026	-0.5044
23	1.0292	0.5995	-0.3935	-0.8022	0.0667	-1.7045	-0.7918	-1.1036	0.3022	1.0953
24	-1.1824	1.2695	-0.7694	-1.1367	-1.3776	0.2233	0.3978	0.6094	-0.6240	-0.5985
25	0.2633	-1.4165	1.0155	1.3986	2.4451	-1.4895	1.6476	0.5600	1.4639	0.4495
26	-0.2898	0.3914	-0.3565	-0.6820	0.8234	-0.6518	1.0142	-0.2614	-0.9183	0.9014
27	0.5268	0.8426	-0.1302	-0.9739	0.0678	-0.4099	-0.6288	0.6093	0.4989	-0.9317
28	1.0089	0.0785	0.3445	-2.3035	0.9081	-0.2625	-0.3125	-1.2488	0.6999	-0.4104
29	-0.5389	-0.3867	-1.7353	-0.4421	-1.3030	0.1701	0.3662	1.2102	-0.5739	-0.4995
30	-2.1502	-1.3589	0.3808	0.1743	-1.0386	0.0660	0.1013	0.1973	-0.7853	-0.3664
31	-0.5669	1.7857	0.0934	0.3592	-1.0683	-1.4864	-1.4372	-0.9318	-0.7486	-0.8363
32	-1.5527	-1.5237	-0.9458	-0.4303	1.1133	-0.0705	1.2097	0.7809	-0.0285	1.3603
33	1.0633	-0.2769	-0.1542	-1.1750	0.9882	0.8631	0.8142	0.2692	1.3912	2.0003
34	0.8899	-0.3766	0.7950	-0.4801	-0.0967	0.1537	1.8660	0.6762	0.9963	0.9811
35	-0.9304	-1.4074	-0.3674	0.5763	-1.6825	1.7376	1.5948	-0.1767	0.4761	0.3082
Etc.	…	…	…	…	…	…	…	…	…	…

Annex 8: Breakpoints

Rating	**AAA**	**AA+**	**AA**	**AA-**	**A+**	**A**	**A-**	**BBB+**	**BBB**	**BBB-**	**BB+**	**BB**	**BB-**	**B+**	**B**	**B-**	**CCC**
AAA	1.646	1.841	1.667	1.793			x										
AA+	-1.283	1.709	1.645	1.783	1.604	1.594											
AA	-1.844	-0.814	1.538	1.773	1.597	1.574	1.668		x	x							
AA-	-2.111	-1.577	-1.044	1.381	1.543	1.559	1.652	1.536									
A+	-2.378	-1.879	-1.528	-0.968	1.193	1.499	1.636	1.524	1.343	1.290							
A	-2.484	-2.016	-2.018	-1.707	-1.063	1.207	1.549	1.502	1.331	1.281				0.931			
A-		-2.206	-2.378	-2.130	-1.813	-1.030	1.113	1.438	1.297	1.245	1.040	0.906					
BBB+		-2.400	-2.716	-2.636	-2.362	-1.757	-1.248	1.039	1.247	1.220	1.027	0.896		0.922			
BBB	x			-2.807	-2.636	-2.130	-1.932	-1.198	0.896	1.147	1.002		1.170			x	
BBB-		x	x	-2.929	-3.195	-2.562	-2.260	-1.905	-1.200	0.791	0.966	0.878	1.158		0.751		
BB+						-2.697	-2.678	-2.137	-1.797	-1.211	0.572	0.815	1.146	0.913	0.739		
BB						-2.759	-2.863	-2.235	-2.062	-1.668	-1.134	0.468	1.134	0.904	0.726		
BB-						-2.834	-2.948	-2.457	-2.370	-2.040	-1.667	-1.213	0.781	0.799	0.714		0.740
B+						-2.929		-2.583	-2.414	-2.232	-1.979	-1.645	-0.868	0.524	0.666	0.951	0.740
B								-2.770	-2.462	-2.342	-2.512	-2.005	-1.520	-1.022	0.322	0.902	0.702
B-									-2.748				-2.010	-1.476	-0.955	0.685	0.595
CCC								-2.863	-2.878	-2.489	x	-2.241	-2.113	-1.858	-1.361	-0.645	0.338
D						x	x	-3.195	-3.090	-2.727		-2.576	-2.175	-2.095	-1.734	-1.274	-0.307

Annex 9: Rating assignment

Default	BMW	ADIDAS	HEI.C	DB	DTE	SZU	Allianz	DP	Daimler	Thyssenkrupp
	A+	AA-	BBB-	BBB+	BBB+	BBB-	AA	A-	A	A-
Scenarios										
1	A+	AA-	BBB-	BBB+	BBB	BB+	AA	A-	A	A
2	A+	AA-	BBB-	BBB	A-	BBB-	AA	A	A+	A-
3	A+	AA-	BBB-	A-	BBB+	BBB-	AA	A-	A	A-
4	AAA	AA-	BBB-	BBB+	BBB+	BBB	AA	A-	A	A-
5	A+	AA-	BBB-	BBB+	A-	BBB-	AA	A-	A	A-
6	A+	AA-	BBB-	AA	AA	BB	AA	A-	A	A-
7	AA-	AA-	BBB-	BB	A-	BBB-	AA	A-	A	A-
8	A+	AA-	BBB-	BBB+	BBB+	BBB-	AA	A-	A	A-
9	A+	AA-	BB+	BBB+	BBB+	BBB-	AA	A-	A	A-
10	AAA	AA-	BBB-	BBB+	BBB+	BBB-	AAA	A-	AAA	AA+
11	A+	AA-	BB+	A-	AA	BBB	AA	A-	A	A-
12	AA-	A+	AA-	AA	AA	A-	AA	A-	AAA	A
13	AA-	AA-	BBB-	BBB+	BBB+	BBB-	AA	A-	AAA	A-
14	A+	AA-	BBB	BBB+	A-	BBB	AAA	A-	A+	A-
15	A	A+	BBB-	BBB+	AA	BBB-	AA	AA+	A+	A-
16	AAA	AA-	BB	BBB+	BBB+	AA-	AA	A-	A	A-
17	A+	AA-	BBB	BBB+	BB-	BBB-	AA	A-	A-	A-
18	A	AA-	BBB-	BBB+	AA	AA-	AA	A	AAA	A
19	A+	AA-	BBB-	BBB	BBB+	BBB-	AA	A-	A	AA+
20	AA	AA-	BBB-	BBB+	BBB+	BBB-	AA	A-	A	A-
21	AA-	AA-	BBB-	BBB+	BBB+	AA-	AA	A-	A	A-
22	A+	AA-	BBB	BBB+	BBB+	BB+	AA+	AA+	A	A-
23	A+	AA-	BBB-	BBB+	BBB+	BB	AA	A-	A	A-
24	A	AA-	BBB-	BBB+	BBB	BBB-	AA	A-	A	A-
25	A+	A+	BBB	A-	AA	BB+	AAA	A-	A+	A-
26	A+	AA-	BBB-	BBB+	BBB+	BBB-	AA	A-	A	A-
27	A+	AA-	BBB-	BBB+	BBB+	BBB-	AA	A-	A	A-
28	A+	AA-	BBB-	BB	BBB+	BBB-	AA	BBB+	A	A-
29	A+	AA-	BB	BBB+	BBB	BBB-	AA	A	A	A-
30	AA-	A+	BBB-	BBB+	BBB+	BBB-	AA	A-	A	A-
31	A+	AAA	BBB-	BBB+	BBB+	BB+	AA-	A-	A	A-
32	A	A+	BBB-	BBB+	A-	BBB-	AA	A-	A	A
33	A+	AA-	BBB-	BBB+	BBB+	BBB	AA	A-	A+	AA+
34	A+	AA-	BBB	BBB+	BBB+	BBB-	AAA	A-	A	A-
35	A+	A+	BBB-	BBB+	BBB	AA-	AA+	A-	A	A-
Etc.	…	…	…	…	….	…	...	…	…	…

Annex 10: Values of bonds by rating and number of pieces

	BMW 500	**ADIDAS** 1000	**HEI.C** 1000	**DB** 1000	**DTE** 1000	**SZU** 1000	**Allianz** 100	**DP** 200	**Daimler** 1000	**Thyssenkrupp** 1000
1	1,115,212	1,089,900	1,131,337	1,066,128	990,355	1,029,711	1,127,644	1,134,657	997,126	1,151,248
2	1,115,212	1,089,900	1,131,337	1,064,252	994,924	1,028,779	1,127,644	1,134,681	999,762	1,151,483
3	1,115,212	1,089,900	1,131,337	1,068,327	992,471	1,028,779	1,127,644	1,134,657	997,126	1,151,483
4	1,116,083	1,089,900	1,131,337	1,066,128	992,471	1,033,554	1,127,644	1,134,657	997,126	1,151,483
5	1,115,212	1,089,900	1,131,337	1,066,128	994,924	1,028,779	1,127,644	1,134,657	997,126	1,151,483
6	1,115,212	1,089,900	1,131,337	1,070,388	997,308	1,019,522	1,127,644	1,134,657	997,126	1,151,483
7	1,115,687	1,089,900	1,131,337	1,049,926	994,924	1,028,779	1,127,644	1,134,657	997,126	1,151,483
8	1,115,212	1,089,900	1,131,337	1,066,128	992,471	1,028,779	1,127,644	1,134,657	997,126	1,151,483
9	1,115,212	1,089,900	1,131,593	1,066,128	992,471	1,028,779	1,127,644	1,134,657	997,126	1,151,483
10	1,116,083	1,089,900	1,131,337	1,066,128	992,471	1,028,779	1,127,733	1,134,657	1,001,370	1,154,547
11	1,115,212	1,089,900	1,131,593	1,068,327	997,308	1,033,554	1,127,644	1,134,657	997,126	1,151,483
12	1,115,687	1,089,270	1,145,680	1,070,388	997,308	1,037,555	1,127,644	1,134,657	1,001,370	1,151,248
13	1,115,687	1,089,900	1,131,337	1,066,128	992,471	1,028,779	1,127,644	1,134,657	1,001,370	1,151,483
14	1,115,212	1,089,900	1,137,296	1,066,128	994,924	1,033,554	1,127,733	1,134,657	999,762	1,151,483
15	1,113,004	1,089,270	1,131,337	1,066,128	997,308	1,028,779	1,127,644	1,138,568	999,762	1,151,483
16	1,116,083	1,089,900	1,119,385	1,066,128	992,471	1,039,428	1,127,644	1,134,657	997,126	1,151,483
17	1,115,212	1,089,900	1,137,296	1,066,128	948,907	1,028,779	1,127,644	1,134,657	996,941	1,151,483
18	1,113,004	1,089,900	1,131,337	1,066,128	997,308	1,039,428	1,127,644	1,134,681	1,001,370	1,151,248
19	1,115,212	1,089,900	1,131,337	1,064,252	992,471	1,028,779	1,127,644	1,134,657	997,126	1,154,547
20	1,115,896	1,089,900	1,131,337	1,066,128	992,471	1,028,779	1,127,644	1,134,657	997,126	1,151,483
21	1,115,687	1,089,900	1,131,337	1,066,128	992,471	1,039,428	1,127,644	1,134,657	997,126	1,151,483
22	1,115,212	1,089,900	1,137,296	1,066,128	992,471	1,029,711	1,127,642	1,138,568	997,126	1,151,483
23	1,115,212	1,089,900	1,131,337	1,066,128	992,471	1,019,522	1,127,644	1,134,657	997,126	1,151,483
24	1,113,004	1,089,900	1,131,337	1,066,128	990,355	1,028,779	1,127,644	1,134,657	997,126	1,151,483
Etc.	…	…	…	…	…	…	…	…	…	…

Annex 11: Probability distribution of the portfolio value (€)

	Values	Frequency	Cumulative frequency	R1	R2
1	8,825,227	3	3	0.01%	0.01%
2	8,901,512	0	3	0.00%	0.01%
3	8,977,798	0	3	0.00%	0.01%
4	9,054,083	0	3	0.00%	0.01%
5	9,130,368	0	3	0.00%	0.01%
6	9,206,653	0	3	0.00%	0.01%
7	9,282,939	0	3	0.00%	0.01%
8	9,359,224	0	3	0.00%	0.01%
9	9,435,509	5	8	0.02%	0.03%
10	9,511,795	0	8	0.00%	0.03%
11	9,588,080	2	10	0.01%	0.03%
12	9,664,365	3	13	0.01%	0.04%
13	9,740,650	23	36	0.08%	0.12%
14	9,816,936	145	181	0.48%	0.60%
15	9,893,221	103	284	0.34%	0.95%
16	9,969,506	0	284	0.00%	0.95%
17	10,045,792	0	284	0.00%	0.95%
18	10,122,077	4	288	0.01%	0.96%
19	10,198,362	0	288	0.00%	0.96%
20	10,274,647	4	292	0.01%	0.97%
21	10,350,933	2	294	0.01%	0.98%
22	10,427,218	70	364	0.23%	1.21%
23	10,503,503	240	604	0.80%	2.01%
24	10,579,789	38	642	0.13%	2.14%
25	10,656,074	194	836	0.65%	2.79%
26	10,732,359	12	848	0.04%	2.83%
27	10,808,644	942	1790	3.14%	5.97%
28	10,884,930	28210	30000	94.03%	100.00%
29	10,961,215	0	30000	0.00%	100.00%
30	11,037,500	0	30000	0.00%	100.00%

Annex 12: Regulatory capital requirement by standard approach under Basel Accord

Basel I	**Rating**	**Nominal value**	**w**	**RWA**	**CR**
BMW	A+	1,000,000	100%	1,000,000	80,000
Adidas	AA-	1,000,000	100%	1,000,000	80,000
HeidelbergCement	BBB-	1,000,000	100%	1,000,000	80,000
Deutsche Bank	BBB+	1,000,000	20%	200,000	16,000
Deutsche Telekom	BBB+	1,000,000	100%	1,000,000	80,000
Suedzucker	BBB-	1,000,000	100%	1,000,000	80,000
Allianz	AA	1,000,000	100%	1,000,000	80,000
Deutsche Post	A-	1,000,000	100%	1,000,000	80,000
Daimler	A	1,000,000	100%	1,000,000	80,000
Thyssenkrupp	A-	1,000,000	100%	1,000,000	80,000
Total	-	-	-	**9,200,000**	**736,000**

Basel II - SA	**Rating**	**Nominal value**	**w**	**RWA**	**CR**
BMW	A+	1,000,000	50%	500,000	40,000
Adidas	AA-	1,000,000	20%	200,000	16,000
HeidelbergCement	BBB-	1,000,000	100%	1,000,000	80,000
Deutsche Bank	BBB+	1,000,000	100%	1,000,000	80,000
Deutsche Telekom	BBB+	1,000,000	100%	1,000,000	80,000
Suedzucker	BBB-	1,000,000	100%	1,000,000	80,000
Allianz	AA	1,000,000	20%	200,000	16,000
Deutsche Post	A-	1,000,000	50%	500,000	40,000
Daimler	A	1,000,000	50%	500,000	40,000
Thyssenkrupp	A-	1,000,000	50%	500,000	40,000
Total				**6,400,000**	**512,000**

Basel III - SA	**Rating**	**Nominal value**	**w**	**RWA**	**CR**
BMW	A+	1,000,000	50%	500,000	52,500
Adidas	AA-	1,000,000	20%	200,000	21,000
HeidelbergCement	BBB-	1,000,000	100%	1,000,000	105,000
Deutsche Bank	BBB+	1,000,000	100%	1,000,000	105,000
Deutsche Telekom	BBB+	1,000,000	100%	1,000,000	105,000
Suedzucker	BBB-	1,000,000	100%	1,000,000	105,000
Allianz	AA	1,000,000	20%	200,000	21,000
Deutsche Post	A-	1,000,000	50%	500,000	52,500
Daimler	A	1,000,000	50%	500,000	52,500
Thyssenkrupp	A-	1,000,000	50%	500,000	52,500
Total				**€ 6,400,000**	**€ 672,000**

Annex 13: Regulatory capital requirement by foundation IRB approach under Basel Accord

Basel II - FIRB	**Rating**	**Nominal value**	**PD**	**LGD**	**R**	**b**	**CR**	**RWA**	**EAD**	**CR**	**RWA**
BMW	A+	1,000,000	0.0142%	48.87%	23.92%	36.46%	0.803%	10.03%	1,000,000	8,025	100,317
Adidas	AA-	1,000,000	0.0044%	48.87%	23.97%	44.63%	0.418%	5.22%	1,000,000	4,179	52,235
HeidelbergCement	BBB-	1,000,000	0.2747%	48.87%	22.46%	19.50%	4.515%	56.44%	1,000,000	45,151	564,392
Deutsche Bank	BBB+	1,000,000	0.2045%	48.87%	22.83%	20.95%	3.860%	48.25%	1,000,000	38,600	482,497
Deutsche Telekom	BBB+	1,000,000	0.2045%	48.87%	22.83%	20.95%	3.860%	48.25%	1,000,000	38,600	482,497
Suedzucker	BBB-	1,000,000	0.2747%	48.87%	22.46%	19.50%	4.515%	56.44%	1,000,000	45,151	564,392
Allianz	AA	1,000,000	0.0024%	48.87%	23.99%	49.17%	0.315%	3.93%	1,000,000	3,145	39,314
Deutsche Post	A-	1,000,000	0.2020%	48.87%	22.85%	21.01%	3.834%	47.93%	1,000,000	38,343	479,283
Daimler	A	1,000,000	0.1075%	48.87%	23.37%	24.30%	2.687%	33.59%	1,000,000	26,870	335,879
Thyssenkrupp	A-	1,000,000	0.2020%	48.87%	22.85%	21.01%	3.834%	47.93%	1,000,000	38,343	479,283
Total										**286,407.08**	**3,580,088**

Basel III - FIRB	Rating	Nominal value	PD	LGD	R	b	CR	RWA	EAD	CR	RWA
BMW	A+	1,000,000	0.0142%	48.87%	0.23915	0.364639	1.053%	13.17%	1,000,000	10,533	131,666
Adidas	AA-	1,000,000	0.0044%	48.87%	0.23974	0.446272	0.548%	6.86%	1,000,000	5,485	68,558
HeidelbergCement	BBB-	1,000,000	0.2747%	48.87%	0.2246	0.194985	5.926%	74.08%	1,000,000	59,261	740,765
Deutsche Bank	BBB+	1,000,000	0.2045%	48.87%	0.22834	0.209523	5.066%	63.33%	1,000,000	50,662	633,277
Deutsche Telekom	BBB+	1,000,000	0.2045%	48.87%	0.22834	0.209523	5.066%	63.33%	1,000,000	50,662	633,277
Suedzucker	BBB-	1,000,000	0.2730%	48.87%	0.22469	0.195286	5.907%	73.84%	1,000,000	59,071	738,382
Allianz	AA	1,000,000	0.0024%	48.87%	0.23986	0.491737	0.413%	5.16%	1,000,000	4,128	51,600
Deutsche Post	A-	1,000,000	0.2020%	48.87%	0.22847	0.210141	5.032%	62.91%	1,000,000	50,325	629,058
Daimler	A	1,000,000	0.1075%	48.87%	0.23372	0.243015	3.527%	44.08%	1,000,000	35,267	440,841
Thyssenkrupp	A-	1,000,000	0.2020%	48.87%	0.22847	0.210141	5.032%	62.91%	1,000,000	50,325	629,058
Total										**375,719**	**4,696,484**